



1 **Revisiting Azimuthally Asymmetric Moist Instability in the**
2 **Outer Core of Sheared Tropical Cyclones**

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19 This study revisits the characteristics and physical processes of the azimuthally
20 asymmetric distribution of moist instability in the outer core of vertically sheared
21 tropical cyclones (TCs) using a numerical model. The results indicate that a downshear-
22 upshear contrast in outer-core conditional instability occurs in the weakly sheared TCs,
23 while an enhanced downshear left-downshear right difference is found in strongly
24 sheared storms. Specifically, lower (higher) conditional instability arises downshear left
25 (right) in the strongly sheared TCs. Downward transports of low-entropy air by
26 convective and mesoscale downdrafts in principal rainbands reduce the equivalent
27 potential temperature (θ_e) in the downshear-left boundary layer, contributing to lower
28 convective available potential energy. Positive horizontal advection of both potential
29 temperature and water vapor by the asymmetric outflow leads to a midlevel maximum
30 of θ_e in the same quadrant. Resultingly, a positive θ_e vertical gradient (thus potential
31 stability) is present in the downshear-left outer core. In the downshear-right quadrant,
32 a lack of convective downdrafts, together with surface fluxes, leads to higher θ_e in the
33 boundary layer. A dry intrusion is found at the middle to upper levels in the downshear-
34 right outer core, and significant negative horizontal advection of water vapor produces
35 low θ_e near the midtroposphere. A negative vertical gradient of θ_e (thus potential
36 instability) in the outer core arises below the downshear-right midtroposphere. The
37 presence of azimuthally asymmetric moist instability is expected to play an important

38 role in fostering and maintaining azimuthally asymmetric convective activity in the
39 outer core of TCs.

40 1. Introduction

41 It has been observationally and theoretically realized that environmental vertical
42 wind shear (VWS) has prominent impacts on tropical cyclone (TC) structure and
43 intensity change. Strong shear is documented to generally inhibit TC intensification
44 (Frank and Ritchie 2001; Riemer et al. 2010; Tang and Emanuel 2010; Gu et al. 2015;
45 Fu et al. 2019), and to force azimuthal asymmetries in convection (Jones 1995; Reasor
46 et al. 2000; Frank and Ritchie 2001; Xu and Wang 2013; Reasor et al. 2013).

47 VWS generally produces an azimuthal wavenumber-1 asymmetry in eyewall
48 convection with highest precipitation in the downshear-left quadrant (Jones 1995; Wang
49 and Holland 1996; Reasor et al. 2000; Frank and Ritchie 2001; Corbosiero and Molinari
50 2002, 2003; Black et al. 2002; Xu and Wang 2013; Reasor et al. 2013; Barnes and
51 Barnes 2014). Airborne Doppler radar observations (DeHart et al. 2014; Wadler et al.
52 2018) demonstrate that convective bursts in the vicinity of the eyewall typically initiate
53 downshear right. As the convective bursts move cyclonically around the TC center, they
54 develop vertically upward, reaching higher elevations in the downshear-left quadrant.

55 Marked asymmetries in convective activity are present in the outer core (roughly
56 outside three times the radius of maximum wind; Wang 2009) of sheared TCs.
57 Corbosiero and Molinari (2002), Stevenson et al. (2014), and Stevenson et al. (2016)
58 examined flash locations in TCs, showing a strong preference for outer rainband flashes
59 in the downshear-right quadrant. Many previous studies have indicated that convective

60 cells within the part of the principal rainband in the downshear-right quadrant tend to
61 collapse as they move into the downwind portion of the band where stratiform clouds
62 become predominant (Hence and Houze 2008; Houze 2010; Didlake and Houze 2013).
63 Li et al. (2017) documented that wavenumber-1 principal rainbands form downshear in
64 sheared TCs, which is the result of the downshear-right convective reinvigoration of
65 inner rainbands after they move outside the inner core. Riemer (2016) revisited the
66 formation mechanism for the wavenumber-1 quasi-stationary band complex in the outer
67 core of sheared TCs. He found that the overlap of regions of high-entropy air and a
68 positive vorticity anomaly in lower layers on the right of the shear vector plays a
69 fundamental role in initiating deep convection associated with the band complex.

70 Lift, instability, and moisture are the necessary conditions for the initiation of
71 deep convection (Sherwood 2000; Schultz et al. 2000). The asymmetric distribution of
72 moist instability occurs in the outer core of sheared TCs, hence possibly accompanying
73 occurrences of asymmetric convection in the outer core. Observations indeed indicate
74 that VWS enables azimuthally asymmetric distributions of moist instability within the
75 TC circulation (Molinari and Vollaro 2008; Molinari and Vollaro 2010; Molinari et al.
76 2012). Molinari and Vollaro (2008) analyzed the combined data of dropsonde
77 soundings and gridded analyses in Hurricane Bonnie (1998), indicative of much larger
78 convective available potential energy (CAPE) values associated with downshear
79 convective cells. They further extended the data to eight hurricanes (Molinari and
80 Vollaro 2010), and a striking downshear-upshear difference in CAPE was found as well,

81 with the average value of downshear CAPE about 60% greater than upshear CAPE for
82 highly sheared storms. Moreover, Molinari et al. (2012) continued to address the CAPE
83 calculation with and without condensate loading, entrainment, and latent heating of
84 fusion based on more than 2000 dropsonde soundings, again confirming the
85 circumstance of larger CAPE in the downshear semicircle within the 400-km radius
86 from the storm center. They proposed that larger downshear CAPE results likely from
87 higher surface moisture due to larger surface fluxes, cooler midlevel temperatures, and
88 a more humid free-troposphere for entraining CAPE. Given the azimuthally
89 asymmetric distribution of conditional instability mentioned above, convective cells are
90 expected to preferentially form and develop in the quadrant where larger conditional
91 instability exists, if air parcels are lifted.

92 Many studies have indicated an asymmetric, shear-induced entropy distribution
93 within the TC boundary layer, and the azimuthally-varying vertical gradient of
94 equivalent potential temperature (θ_e) possibly implies an azimuthal asymmetry in
95 potential instability (Schultz and Schumacher 1999; Rosenow et al. 2018) within the
96 TC circulation. For instance, Zhang and Rogers (2019) discussed the impact of the
97 boundary layer structure on Hurricane Earl (2010)'s rapid intensification based on
98 numerical simulations, illustrating lower (higher) θ_e predominantly downshear left
99 (downshear right) both in the inner and outer cores (their Fig. 11). If significant
100 potential instability arises in an individual quadrant, deep convection possibly develops
101 there in the presence of layer lifting once the TC outer-core circulation interacts with a

102 density current, front, or broad mountain range.

103 The studies mentioned above suggest a visible downshear-upshear contrast in
104 moist instability in the outer core of sheared TCs, commonly with relatively higher
105 instability in the downshear semicircle or sometimes particularly in the downshear-right
106 quadrant. The release of azimuthally asymmetric instability is expected to play a salient
107 role in generating and maintaining azimuthally asymmetric convective structures in the
108 outer core. Moreover, the convective activity associated with azimuthally asymmetric
109 moist instability in the outer core may have important effects on TC structure and
110 intensity change. For instance, convection in spiral outer rainbands have been
111 documented in previous studies to have marked impacts on TC intensity in various ways,
112 usually suppressing TC intensification or weakening a TC (Barnes et al. 1983; Powell
113 1990a,b; Wang 2009; Li and Wang 2012a; Fu et al. 2019). Convective-scale downdrafts
114 forced by convective cells in outer rainbands could bring low-entropy air downward
115 into the boundary layer (Barnes et al. 1983; Powell 1990a,b; Hense and Houze 2008;
116 Didlake and Houze 2009; Li and Wang 2012b). When such low-entropy air is further
117 transported radially inward into the inner core of the TC, intensification would be
118 suppressed, and the TC can weaken (Barnes et al. 1983; Powell 1990a,b; Li and Wang
119 2012a; Fu et al. 2019). Wang (2009) documented that diabatic heating produced by the
120 convection in outer rainbands tends to lower the local near-surface pressure, thus
121 reducing the horizontal pressure gradient across the radius of maximum wind and
122 limiting TC intensity. Therefore, the azimuthally asymmetric distribution of moist

123 instability in the outer core is likely to significantly modulate the convective activity of
124 outer rainbands of sheared TCs, accordingly influencing the abovementioned
125 detrimental role of outer rainbands in TC intensity.

126 Certain aspects regarding the azimuthally asymmetric moist instability still
127 deserve further illumination as the asymmetric distribution of the instability likely
128 dictates asymmetric convective activity in the outer core. For example, how is the
129 degree of the asymmetry in outer-core conditional instability dependent on VWS
130 magnitude? Does azimuthally asymmetric convective instability occur in the outer core
131 of sheared TCs? In addition, fundamental physical processes giving rise to the
132 occurrence of the azimuthally asymmetric moist instability in VWS still need further
133 investigation. In this study, we thus revisit the traits of outer-core moist instability in
134 sheared TCs, glean insights into the causes for the asymmetric distribution of the
135 instability. Observations have shown that VWS associated with TCs generally has a
136 wide variety of magnitudes when TCs are embedded within different environmental
137 circulations (Rios-Berrios and Torn 2017). High-resolution numerical experiments will
138 be conducted here to examine the characteristics of outer-core moist instability in TCs
139 under environmental VWS with different magnitudes.

140 The present paper is organized as follows. In section 2, the model used and
141 experimental design are outlined. The azimuthally asymmetric characteristics of
142 conditional and potential instabilities, and physical processes modulating the
143 instabilities, are present in sections 3 and 4, respectively. Section 5 summarizes the

144 conclusions from the study.

145 **2. Model and experimental design**

146 The fully compressible, nonhydrostatic TC model, version 4 (TCM4), is used in
147 this study, and a full description of TCM4 can be seen in Wang (2007). The physical
148 parameterizations employed in TCM4 are summarized in Table 1. TCM4 has been used
149 to successfully model a wide variety of TC structures, such as annular hurricanes (Wang
150 2008) and spiral rainbands (Wang 2009; Li and Wang 2012a,b; Li et al. 2017), and to
151 investigate fundamental dynamics regarding TC structure and intensity change (Wang
152 and Xu 2010; Fudeyasu and Wang 2011; Xu and Wang 2013; Li et al. 2014, 2015; Heng
153 and Wang 2016).

154 In the present simulations, quadruply nested domains are employed with two-way
155 interactive nesting, with domain sizes of 12960 km \times 12960 km (D1), 2268 km \times 2268
156 km (D2), 972 km \times 972 km (D3), and 624 km \times 624 km (D4). The grids have 32 vertical
157 levels, and the horizontal grid intervals are 54, 18, 6, and 2 km, respectively. No
158 cumulus parameterization is employed, even in the two outermost domains, as
159 convection occurs mainly within the inner core of the modeled cyclone. The model is
160 run on an f -plane at 18°N over the ocean with a fixed sea surface temperature of 29°C.
161 An initial vortex has a maximum tangential wind velocity of 20 m s⁻¹ at the 90-km
162 radius near the surface, decreasing sinusoidally with pressure to zero at 100 hPa. The
163 initial thermodynamic profile of the unperturbed model atmosphere is derived from the

164 moist-tropical sounding of Dunion (2011).

165 After a 60-h spinup ($T=0$ h assigned at this time), the minimum surface pressure
166 of the simulated TC drops to approximately 965 hPa (Fig. 1), with a radius of maximum
167 wind of 30 km and an evident warm core with a temperature anomaly exceeding 7 K
168 near $z = 8$ km (not shown). At this time, easterly shears of 5, 15, and 25 m s^{-1} are
169 introduced, respectively, with the zonal wind velocity increasing from 0 m s^{-1} at about
170 $z = 1.5$ km to 5, 15, and 25 m s^{-1} at about $z = 13.5$ km and remaining constant above
171 (see the inset in Fig. 1), respectively. Subsequently, the simulations continue for 48
172 hours. The sensitivity simulations of 5-, 15-, and 25- m s^{-1} shears are labeled SH05,
173 SH15, and SH25, respectively. The 25th and 75th percentiles of the global distribution
174 (4.5 and 11.0 m s^{-1} , respectively) of VWS were defined in Rios-Berrios and Torn (2017)
175 as the lower and upper bound of moderate shear. Figure 4 in Rios-Berrios and Torn
176 (2017) shows that the 25th and 75th percentiles of the shear relevant to North Atlantic
177 hurricanes are subtly larger than those for the global TCs, with values of 5 and 12 m s^{-1} ,
178 respectively. Given the North Atlantic moist-tropical sounding of Dunion (2011) used
179 as the initial thermodynamic profile of the model atmosphere, SH05, SH15, and SH25
180 imply the scenario of a TC in weak, strong, and extreme environmental vertical shears,
181 respectively, especially for North Atlantic hurricanes. The experiment settings are
182 identical to those in Li and Fang (2018).

183 Figure 1 shows the intensity evolution of TCs modeled in the three experiments.
184 After a shear of 5 m s^{-1} is introduced in SH05, the storm still intensifies, with the

185 minimum sea-level pressure dropping to about 922 hPa at 48 h. The intensity change
186 substantiates that, to an extent, a TC under weak VWS can still intensify (Wang et al.
187 2015; Rios-Berrios and Torn 2017). For SH15, the simulated storm tends to weaken,
188 exhibiting intensity oscillations. Such intensity oscillations are possibly associated with
189 the quasi-periodic outer rainband activity (Li and Wang 2012a), or the vortex tilt and
190 succeeding realignment (Reasor et al. 2004; Jones et al. 2009), which needs further
191 investigation. As a very strong shear of 25 m s^{-1} is imposed in SH25, the TC rapidly
192 weakens (Fig. 1), and then the vortex circulation becomes indistinct. Therefore, we will
193 discuss only the first 12-h for SH25 thereafter.

194 **3. Azimuthally asymmetric distribution of conditional instability in the outer** 195 **core**

196 *a. CAPE and reflectivity*

197 CAPE (Moncrieff and Miller 1976), roughly defined as the vertically integrated
198 buoyancy of adiabatically lifted air, is generally used to evaluate the degree of
199 conditional instability (Schultz et al. 2000). CAPE corresponds theoretically to
200 convective activity (Weisman and Klemp 1982) and can be used to estimate the upper
201 bound of the theoretical maximum updraft velocity. Therefore, it regularly acts as one
202 of the environmental ingredients for moist convection (Emanuel 1994; Rasmussen and
203 Blanchard 1998). As mentioned in the introduction, the asymmetric distribution of
204 CAPE has been observed in sheared TCs. Such an asymmetry is revisited in this

205 subsection. CAPE is defined as:

$$206 \quad \text{CAPE} = \int_{LFC}^{EL} g \frac{T_v - T_{ve}}{T_{ve}} dz, \quad (1)$$

207 where T_v is the virtual temperature of the parcel; T_{ve} the virtual temperature of the
208 environment; g the gravitational acceleration; z the vertical height; LFC the level of
209 free convection; EL the equilibrium level; and the overbar represents the mean through
210 the depth, dz . The values of CAPE in this study are calculated from vertical profiles and
211 assume that an undiluted parcel is characterized by the mean humidity and temperature
212 in the lowest 500 m.

213 Figure 2 depicts the horizontal distributions of modeled reflectivity at $z = 3$ km,
214 superimposed by CAPE values. Consistent with prior findings, deep-layer VWS
215 produces convective asymmetries in the inner core (approximately inside a 100-km
216 radius here), with strongest convection in the downshear-left quadrant (Figs. 2a and 2b).
217 Another prominent feature in SH15 is the preferred existence of principal rainbands
218 (Willoughby et al. 1984; Willoughby 1988) outside the inner core. Such outer rainbands
219 tend to arise downshear, and their formation is related closely to the convective
220 reinvigoration of downshear inner rainbands (Li et al. 2017). Although a downshear
221 outer rainband appears in SH05 at 24 h (Fig. 2a), visible outer rainbands also exist
222 frequently in other quadrants (not shown). The preferentiality of wavenumber-1
223 principal rainbands is hence less significant in weak shear, although the convection in
224 downshear outer rainbands seems to be more active (Fig. 2a) than that in other

225 quadrants. As the shear increases up to 25 m s^{-1} (namely in experiment SH25), a
226 reflectivity structure like a mesoscale convective system is positioned downshear left
227 outside the inner core (Fig. 2c).

228 *b. An overview of azimuthally asymmetric distribution of outer-core CAPE*

229 CAPE within the TC circulation is characterized by azimuthal asymmetries in the
230 experiments. In the experiment with weak shear (namely SH05), higher CAPE is
231 located downshear at 24 h, and lower CAPE values occur upshear, with the lowest value
232 upshear right outside a radius of 100 km (Fig. 2a). Such an asymmetry is also shown in
233 the time-azimuth plot of CAPE radially averaged between 100 and 300 km (Fig. 3a).
234 About 7 hours after the shear is imposed, a downshear-upshear contrast of CAPE occurs,
235 with higher values in the downshear semicircle and lower values in the upshear
236 semicircle (Fig. 3a). Molinari and Vollaro (2010) indicated that the mean values of
237 CAPE in the downshear semicircle are comparable to those in the upshear semicircle
238 for $VWS < 10 \text{ m s}^{-1}$. The discordance between their results and the current study is
239 because possibly the dropsonde soundings used in Molinari and Vollaro (2010)
240 stemmed from eight hurricanes within diverse thermodynamic environments and were
241 located mostly in the inner cores.

242 As the shear increases to 15 and 25 m s^{-1} , the wavenumber-1 asymmetric CAPE
243 structure appears more pronounced, with higher (lower) values shifting to the right (left)
244 of the shear vector (Figs. 2b and 2c), reminiscent of the results in Molinari et al. (2012).

245 The value of CAPE in the downshear right quadrant is higher than that in other
246 quadrants (Figs. 3b and 3c). Figures 3b and 3c further illustrate that CAPE in the
247 downshear-left outer core is much lower than in other quadrants, particularly in the
248 middle and downwind sectors of this quadrant. Also, higher convective inhibition (CIN)
249 occurs upshear left (Figs. 3e and 3f). A similar asymmetric distribution of outer-core
250 CAPE was observed in Hurricane Earl (2010) in Stevenson et al. (2014). Their Fig. 11
251 depicted the lowest CAPE values in the downshear-left outer core when Earl was
252 experiencing VWS of approximately 8.5 m s^{-1} .

253 All in all, wavenumber-1 azimuthally asymmetric CAPE in the outer core of the
254 sheared TCs is reproduced well in the numerical simulations, with higher CAPE located
255 downshear for weak shear and highest values downshear right for strong shear. We will
256 examine the causes of the asymmetric CAPE in SH15 and SH25 in the following
257 subsections, mainly focusing on the downshear-right and downshear-left quadrants
258 where CAPE values are vastly contrasting.

259 *c. Higher CAPE in the downshear-right outer core*

260 Molinari et al. (2012) pointed out that more considerable near-surface humidity,
261 likely due to more significant surface fluxes in the downshear semicircle, plays a critical
262 role in the existence of larger CAPE in that semicircle, although the near-surface
263 temperatures were comparable downshear and upshear. The present simulations display
264 that azimuthally asymmetric distributions of both near-surface temperatures and

265 humidity are more striking in the strong and extreme shear environments (Figs. 4c, 4d,
266 4e, and 4f) than in the weak shear environment (Figs. 4a and 4b). In particular, much
267 higher values of near-surface potential temperatures and humidity occur right-of-shear
268 in SH15 and SH25 (Figs. 4c, 4d, 4e, and 4f), with maxima in the downshear-right
269 quadrant. The difference, particularly in the downshear-upshear temperature contrast
270 between Molinari et al. (2012) and the current study, is possibly owing to soundings in
271 multiple TCs in Molinari et al. (2012), the storms most of which (> 85%) were in weak
272 and moderate VWS and were likely embedded in different thermodynamic
273 environments. As noted in Zhang et al. (2013) and Nguyen et al. (2017), although
274 downshear-left downdrafts initially deposit drier air into the boundary layer,
275 accumulated moistening via surface fluxes makes the boundary layer more humid right-
276 of-shear as the air is advected cyclonically. There is thus much higher equivalent
277 potential temperature (θ_e) air in the downshear-right outer core in SH15 and SH25 (Figs.
278 7b and 7c; discussed later), producing larger CAPE there (Figs. 3b and 3c).

279 The soundings in Molinari et al. (2012) show that there is the largest downshear-
280 upshear temperature difference in the mid-troposphere, with colder air in the downshear
281 semicircle, also contributing to the larger CAPE in the same semicircle. More
282 significant midlevel temperature differences are also observed in the outer core in SH15
283 and SH25 (Figs. 5c and 5e), compared to SH05 (Fig. 5a). The mid-tropospheric air in
284 the downshear-right quadrant is colder than that in the upshear-right quadrant (Figs. 5c
285 and 5e). As a result, outer-core CAPE is the highest downshear right in SH15 and SH25.

286 Figure 5d shows a weak intrusion of dry air at midlevels in the downshear-right outer
287 core near a radius of 200 km in SH15. McCaul (1987) and Curtis (2004) hypothesized
288 that dry air, which is ingested in the midtroposphere, would lead to an increase in
289 evaporative cooling and steepen the lapse rate (thus a local increase in conditional
290 instability). However, Fig. 6b suggests little latent cooling at midlevels in the
291 downshear-right quadrant in SH15, and the strengthening of evaporative cooling
292 associated with dry intrusions (McCaul 1987; Curtis 2004) seems not to be discerned.
293 Indeed the occurrence of cooler potential temperatures at midlevels in the downshear-
294 right quadrant (Figs. 5c) results from adiabatic cooling by convective-scale updrafts,
295 which will be discussed later.

296 As mentioned in the introduction, Corbosiero and Molinari (2002), Stevenson et
297 al. (2014), and Stevenson et al. (2016) revealed a strong downshear-right favorableness
298 of lightning flashes in the outer rainbands. They stated that such a preference might be
299 attributed to convection in the stationary band complex (Willoughby et al. 1984).
300 Although the relationship between the downshear-right favorableness of flashes and the
301 azimuthally asymmetric distribution of outer-core CAPE is not the subject of this study,
302 we may make the following hypothesis. Li et al. (2017) demonstrated that
303 environmental VWS preferably forces outer rainbands to form downshear. After the
304 inner rainbands associated with vortex Rossby waves or triggered by flow deformation
305 move outside the rapid filamentation zone, they convectively reinvigorate to form outer
306 rainbands in the downshear semicircle because of not only reduced deformation but

307 also enhanced CAPE. With the development of the downshear outer rainbands, their
308 upwind and middle sectors are located mainly in the downshear-right quadrant where
309 CAPE is larger than in other quadrants, as revealed above. Convection with enhanced
310 updrafts is likely fostered in the upwind and middle sectors of the rainbands (Hence
311 and Houze 2008; Houze 2010), which facilitates the preference of flashes as observed
312 in Corbosiero and Molinari (2002), Stevenson et al. (2014), and Stevenson et al. (2016).

313 *d. Lower CAPE in the downshear-left outer core*

314 Figures 3b and 3c indicate CAPE is much lower in the downshear-left quadrant
315 than in other quadrants in SH15 and SH25. As noted above, the strong VWS yields a
316 downshear preference of outer rainbands (namely principal rainbands; Li et al. 2017).
317 The upwind, middle, and downwind portions of a well-developed outer rainband are
318 accordingly characterized by nascent convective cells in the downshear-right quadrant,
319 mature cells in the downshear-left quadrant, and stratiform clouds in the upshear-left
320 quadrant, respectively (Hence and Houze 2008). Within a principal rainband, cooling
321 due to rainwater evaporation can trigger convective-scale downdrafts close to the
322 updraft core (Barnes et al. 1983; Powell 1990a,b; Didlake and Houze 2009; Li and
323 Wang 2012b) and mesoscale subsidence in the downwind stratiform sector of the
324 rainband (Riemer et al. 2010; Didlake and Houze 2013). One critical role of such
325 sinking motion is to transport low-entropy air downward (Barnes et al. 1983; Powell
326 1990a,b; Li and Wang 2012a).

327 Figure 6 depicts time-height cross sections of latent heating rate radially averaged
328 between 100 and 300 km for the experiments, with the stippling denoting where sinking
329 motion is located. In SH15 and SH25, relatively shallower, but larger, cooling is present
330 in the downshear-left boundary layer (Figs. 6e and 6f), resulting mainly from the
331 evaporation of rainwater in subsaturated air beneath convective clouds, and leading to
332 low-level, convective-scale downdrafts (Didlake and Houze 2009; Li and Wang 2012b).
333 In the upshear-left quadrant, deeper, but less, cooling dominates low to midlevels in
334 SH15 (Fig. 6h), caused by rainwater evaporation underneath stratiform clouds (Fig. 2b;
335 Riemer et al. 2010; Didlake and Houze 2013). Such low-level cooling is not apparent
336 in SH25 (Fig. 6i) because stratiform precipitation is absent in the upshear-left quadrant
337 in that extreme shear environment (Fig. 2c). As a result, cooling associated with the
338 convective-scale downdrafts yields potential temperature minima between 100 and 200
339 km in the downshear-left quadrant (Figs. 4c and 4e) due to the evaporation of more
340 precipitation. This evaporative cooling associated with the convective-scale downdrafts
341 causes weaker conditional instability in the downshear-left quadrant. In the remaining
342 quadrants, there is a lack of cooling relevant to intense sinking motion (Figs. 6b, 6c, 6k,
343 and 6l). In contrast, no azimuthal preferentiality of evaporative cooling is observed in
344 SH05 (Figs. 5a, 5d, 5g, and 5j) because weak VWS does not tend to initiate visible
345 wavenumber-1 principal rainbands in this experiment.

346 In SH15 and SH25, the humidity between 100 and 200 km near the surface is much
347 lower in the downshear-left quadrant than in other quadrants (Figs. 4d and 4f), resulting

348 from the evaporationally induced downdrafts that transport drier air aloft downward
349 (discussed later). Together with the potential temperature minima, much lower
350 boundary-layer θ_e occurs downshear left (Figs. 7b and 7c). Although colder mid-
351 tropospheric air occurs in the downshear-left quadrant (Figs. 5c and 5e), lower CAPE
352 is still observed therein (Figs. 3b and 3c). This implies that the existence of much lower
353 boundary-layer θ_e accounts mainly for the lower CAPE in the downshear-left
354 quadrant.

355 Many previous studies have indicated that convective cells within the TC principal
356 rainband tend to collapse as they move into the downwind portion of the band where
357 broad stratiform clouds become predominant (Hence and Houze 2008; Houze 2010;
358 Didlake and Houze 2013). Why do the cells weaken therein? Two reasons are
359 hypothesized. One is the increased filamentation effect because the convection tracks
360 more radially inward when cyclonically moving along the spiral rainband. The other is
361 the visible decrease in conditional instability discussed above, which is just located in
362 the middle and downwind sectors of the downshear-left quadrant. As the well-
363 developed convective cells move more downwind, they thus tend to transition into
364 stratiform clouds.

365 **4. Azimuthally asymmetric distribution of potential instability in the outer core**

366 *a. An overview of the azimuthally asymmetric distribution of the θ_e vertical gradient*

367 Figure 7 shows the time-quadrant distributions of θ_e in the three experiments for

368 various heights, radially averaged between 100 and 300 km in different vertical layers.
369 In SH05, there are noticeable downshear-upshear differences in θ_e vertically averaged
370 between $z = 0.1$ and 0.96 km, particularly during 6–26 h and 37–48 h (Fig. 7a), with
371 lower values in the upshear semicircle and higher values in the downshear semicircle.
372 The averaged midlevel θ_e , whether it is downshear or upshear, is mostly lower than
373 that in the boundary layer (Fig. 7d). Such a negative θ_e vertical gradient thus indicates
374 potential instability in the outer core at low to midlevels. At upper levels, positive θ_e
375 vertical gradients are present in all the quadrants (Figs. 7d and 7g).

376 As the magnitude of shear increases, the azimuthal asymmetry in outer-core θ_e
377 becomes sharper. For example, higher θ_e averaged between $z = 0.1$ and 0.96 km in
378 SH15 occurs right-of-shear, with peak values > 350 K downshear right (Fig. 7b). The
379 minimum values of θ_e averaged between $z = 0.1$ and 0.96 km in SH15 are lower,
380 compared to those in SH05, with the lowest θ_e values shifting to the downshear-left
381 quadrant (Fig. 7b). The lower values of θ_e vertically averaged between $z = 4.3$ and 5.0
382 km exists on the right side of the VWS and higher θ_e is located left-of-shear, with
383 maximum values > 343 K in the downshear-left quadrant (Fig. 7e). Similar asymmetric
384 patterns are seen in SH25, with higher θ_e occurring in the downshear-left quadrant at
385 mid- and upper-levels and occupying an azimuthally-compact area during 6 to 12 h of
386 simulation (Figs. 7f and 7i). The patterns of θ_e vertically averaged between $z = 9.6$ and
387 10.6 km in SH15 and SH25 resemble those in the midtroposphere, with the highest
388 values (> 347 K) in the downshear-left quadrant (Figs. 7h and 7i).

389 The above results thus show a notable negative vertical gradient of θ_e at low to
 390 midlevels in the downshear-right outer core in SH15 and SH25 (Figs. 7b, 7c, 7e, and
 391 7f), suggestive of a potentially unstable environment. Above midlevels, θ_e increases
 392 with height in the same quadrant, indicative of the presence of potential stability. In the
 393 downshear-left outer core, there is a positive θ_e vertical gradient throughout the
 394 troposphere, demonstrating potential stability in that quadrant.

395 To further investigate the processes associated with the θ_e potential instability
 396 characteristics, θ_e budgets are conducted. The tendency equation for θ_e in TCM4 is

$$397 \quad \frac{\partial \theta_e}{\partial t} = -\mathbf{V}_3 \cdot \nabla_3 \theta - \frac{L}{C_p \pi} \mathbf{V}_3 \cdot \nabla_3 q_v + D_\theta + F_\theta + \frac{L}{C_p \pi} D_{q_v} + \frac{L}{C_p \pi} F_{q_v} + H_\theta, \quad (2)$$

398 where $\mathbf{V}_3 \cdot \nabla_3 = u(\partial/\partial x) + v(\partial/\partial y) + w(\partial/\partial z)$, with u being the zonal wind, v
 399 the meridional wind, and w the vertical wind. In addition, θ , q_v , L , C_p , π , D_θ , F_θ ,
 400 D_{q_v} , F_{q_v} , and H_θ denote the potential temperature, water vapor mixing ratio, latent
 401 heat, specific heat at constant pressure, Exner function, horizontal diffusion of potential
 402 temperature, vertical mixing of potential temperature including surface fluxes,
 403 horizontal diffusion of water vapor mixing ratio, vertical diffusion of water vapor
 404 mixing ratio, and dissipative heating, respectively. The detailed formulation of (2) can
 405 be found in Yang et al. (2007) and Li and Wang (2012a). The first two terms on the
 406 right side of (2) are the three-dimensional advective contributions of θ and q_v to the
 407 θ_e tendency, respectively. The remaining terms on the right are contributions by
 408 diabatic processes.

409 Since the azimuthally asymmetric distribution of θ_e and potential instability is
410 more notable in highly sheared TCs, particularly in the downshear semicircle, we
411 further investigate in the following subsections the associated physical processes in
412 these quadrants in SH15 and SH25 through the θ_e budgets.

413 *b. The θ_e vertical gradient in the downshear-right outer core*

414 Figure 8 displays the time-height cross sections of horizontal advection, vertical
415 advection, diabatic processes, and total θ_e tendencies in SH15, which are radially
416 averaged between 100 and 300 km in the downshear-right and downshear-left quadrants.
417 Note that, although the total θ_e tendencies look somewhat noisy (Figs. 8d and 8h),
418 some of the marked characteristics can still be discerned, which will be elaborated
419 below.

420 Figures 8b and 8c show that the vertical advection and diabatic processes (mainly
421 due to the surface fluxes, F_θ and F_{qv}) contribute to the positive total θ_e tendency
422 predominant below $z = 3$ km in the downshear-right quadrant, particularly during 0–30
423 h (Fig. 8d), although they are partly counteracted by the negative contribution of
424 horizontal advection (Fig. 8a). Consequently, the θ_e value below $z = 3$ km increases
425 in the downshear-right quadrant in SH15 during 0–30 h. For instance, the downshear-
426 right θ_e value averaged within the boundary layer significantly increases during that
427 time and is much higher than in other quadrants (Fig. 7b).

428 Negative horizontal advection between $z = 4.5$ and $z = 11$ km prevails particularly

429 during 3–39 h (Fig. 8a), notwithstanding the positive vertical advection above $z = 7$ km
430 (Fig. 8b). This negative horizontal advection contributes mainly to the negative θ_e
431 tendency predominant between $z = 4.5$ and $z = 10$ km particularly before 24 h in SH15
432 (Fig. 8d). Figure 9 shows time-height cross sections of mean contributions by the
433 horizontal and vertical advection of θ and q_v to the θ_e tendency in SH15. Weak
434 horizontal advection of θ is present in upper layers (e.g., 9.6–10.6 km; Fig. 9a),
435 resulting from the outer-core asymmetric wind vectors in the downshear-right quadrant
436 approximately parallel with the isotherms (Fig. 10c). There exists a pronounced dry air
437 slot between 150- and 250-km radii in the downshear-right quadrant in upper layers
438 (Fig. 10d). Note that, comparatively, no dry tongue is found in upper layers in SH05
439 (Fig. 10b). Therefore, this upper-level dry tongue in SH15 results likely from the dry
440 intrusion by the enhanced upper-layer TC-relative outflow. A weaker dry intrusion in
441 the midtropospheric outer core is also found downshear right (Fig. 5d), as noted in
442 subsection 3b. Therefore, the horizontal advection of q_v averaged in the downshear-
443 right quadrant in SH15 becomes predominantly negative between $z = 4.5$ and $z = 11$
444 km during most of the 48-h simulation time (Fig. 9b). The horizontal advective
445 contribution to the θ_e tendency between $z = 4.5$ and $z = 11$ km is thus mostly negative
446 in that quadrant particularly during 3–39 h (Fig. 8a), responsible for the negative total
447 θ_e tendency between $z = 4.5$ and $z = 10$ km particularly before 12 h (Fig. 8d), as
448 mentioned above. Consequently, the outer-core θ_e values in both middle and upper
449 layers in SH15 subtly decrease in the downshear-right quadrant before 12 h (Fig. 7h).

450 The above analysis indicates that the negative θ_e vertical gradient (hence
451 potential instability) below the midtroposphere in the downshear-right outer core (Figs.
452 7b and 7e) results from higher θ_e values relevant to the positive θ_e tendency below
453 $z = 3$ km and lower θ_e values associated with the negative θ_e tendency between $z =$
454 4.5 and $z = 11$ km in that quadrant (Fig. 8d). Although the outer-core θ_e value at upper
455 levels in the downshear-right quadrant decreases particularly before 24 h (Fig. 7h) due
456 to the negative total θ_e tendency, it is still higher than that at midlevels in the same
457 quadrant (Fig. 7e). There thus exists potential stability above the midtroposphere in the
458 downshear-right quadrant in SH15.

459 As the VWS is increased up to 25 m s^{-1} (namely in SH25), the general
460 characteristics of the θ_e budget in the outer core on the right of VWS resemble those
461 in SH15. For instance, positive θ_e tendencies prevail beneath the downshear-right
462 midtroposphere (Fig. 11d), yielding a marked increase in θ_e in the downshear-right
463 boundary layer (Fig. 7c). In contrast, negative θ_e tendencies abound above the
464 midlevels (Fig. 11d), making the upper-level θ_e value decrease in the downshear-right
465 quadrant in SH25 (Fig. 7i). Positive horizontal advection of θ is observed mostly in
466 the downshear-right troposphere, and a maximum that is larger than in the downshear-
467 right quadrant in SH15 (Fig. 9a) occurs around $z = 4\text{--}5$ km (Fig. 12a). Negative
468 horizontal advection of q_v primarily dominates in the same quadrant, also with a
469 midlevel minimum (Fig. 12b) that is much smaller than in SH15 (Fig. 9b). As a result,
470 a downshear-right positive (negative) horizontal advection contribution occurs below

471 (above) approximately $z = 4$ km, mainly after 4 h in SH25 (Fig. 11a). In addition, the
472 positive vertical advective contribution of q_v surpasses the negative vertical advective
473 contribution of θ (Figs. 12c and 12d), leading to the predominant positive vertical
474 advection below $z = 3$ km (Fig. 11b). Along with the positive contribution of surface
475 fluxes (Fig. 11c), positive θ_e tendencies thus occur below the midlevels in the
476 downshear-right quadrant (Fig. 11d), contributive to the increase in θ_e and the higher
477 boundary-layer θ_e value in that quadrant (Fig. 7c). The positive θ_e tendencies below
478 the downshear-right midtroposphere in SH25 (Fig. 11d) are larger than those in SH15
479 (Fig. 8d), making the boundary-layer θ_e value in the downshear-right quadrant
480 increase more rapidly in SH25 than in SH15 during the first 12-h simulation (Figs. 7b
481 and 7c). The negative horizontal advective contribution above $z = 4$ km after 4 h mainly
482 produces negative θ_e tendencies above the same altitude (Fig. 11d), resulting in a
483 significant decrease in θ_e and making the θ_e value near $z = 5$ km lower in the
484 downshear-right quadrant than in other quadrants (Fig. 7f). The downshear-right
485 negative θ_e tendencies near $z = 5$ km are smaller in SH25 (Fig. 11d) than those in
486 SH15 (Fig. 8d), leading the θ_e at downshear-right midlevels to decrease more rapidly
487 in SH25 than in SH15 (Figs. 7e and 7f).

488 The existence of significant potential instability below midlevels in the
489 downshear-right outer core of TCs simulated in SH15 and SH25 implies that continuous
490 forcing is required to convert the potential instability into actual instability in that
491 quadrant. It is hence anticipated that deep convection is generated in the downshear-

492 right outer core of a highly sheared TC in the presence of layer lifting by a density
493 current, frontal surface, mesoscale mountain range, and frictional convergence
494 associated with the Ekman pumping due to the outer vortex tilt (Riemer et al. 2010).
495 For example, Hill et al. (1966) pointed out that midlevel dry air intrusions increase the
496 convective instability by reducing humidity and thus θ_e aloft, contributing to
497 landfalling hurricane tornado outbreaks. Although several studies (e.g., McCaul 1987;
498 Curtis 2004) hypothesized midlevel dry intrusions possibly increase the conditional
499 instability by enhancing evaporative cooling and thereby steepening the lapse rate, the
500 lack of latent cooling around midlevels in the downshear-right outer core in SH15 (Fig.
501 6b) where a weak, dry intrusion forced by the VWS occurs (Fig. 5d) indicates such an
502 evaporative cooling effect is likely limited. The specific relationship between the
503 azimuthally asymmetric potential instability and convective occurrences in the outer
504 core is not addressed here because it is beyond the scope of this study, but it is worth
505 further investigation based on observations and numerical simulations.

506 *c. The θ_e vertical gradient in the downshear-left outer core*

507 The θ_e budget averaged in the downshear-left outer core of the TC simulated in
508 SH15 is first examined. A positive θ_e vertical gradient is seen throughout the whole
509 troposphere in the downshear-left outer core of the TC simulated in SH15 (Figs. 7b, 7e,
510 and 7h), and a potentially stable environment thus exists in that quadrant. Negative θ_e
511 tendencies are predominant in the boundary layer at some times (e.g., 0–20 h and 28–
512 38 h; Fig. 8h), resulting in decreases in θ_e and making the θ_e value within the

513 boundary layer relatively lower in the downshear-left quadrant than in other quadrants
514 (Fig. 7b). The θ_e tendency maxima near the midtroposphere (Fig. 8h) become visible
515 after approximately 3 h, and are primarily due to the presence of a shallow layer of
516 enhanced positive horizontal advection at midlevels (Fig. 8e), along with positive
517 vertical advection (Fig. 8f). As a result, there is an increase in θ_e through
518 approximately 30 h and the relatively higher θ_e value in the downshear-left
519 midtroposphere in SH15, compared to other quadrants (Fig. 7e).

520 The strengthened positive midlevel horizontal advection in the downshear-left
521 quadrant aforementioned (Fig. 8e) is due to horizontal advective contributions of both
522 θ and q_v in the same quadrant (Figs. 9e and 9f). As shown in Figs. 5c and 5d,
523 asymmetric outflow, although relatively weak, prevails in the downshear-left quadrant
524 at midlevels in SH15. As θ and q_v associated with the healthy convection are higher
525 in the inner-core region than in the outer core, the asymmetric outflow transports higher
526 θ and q_v radially outward, leading to downshear-left enhancements of horizontal
527 advection of θ and q_v . Note that the q_v value, which is larger in the downshear-left
528 quadrant than in other quadrants (Fig. 5d), is due to not only the horizontal advection
529 mentioned above but also the vertical moisture transport in the downshear-left quadrant
530 (Fig. 9h). Figure 9 also indicates that, although the vertical advective contributions of
531 θ and q_v counteract each other, net positive vertical advective contributions of the
532 two quantities exist downshear left between $z = 1$ km and $z = 5.5$ km (Fig. 8f). As a
533 result, positive θ_e tendency maxima arise in the downshear-left midtroposphere in

534 SH15 (Fig. 8h).

535 As noted in Section 3c, the potential temperatures in the midtropospheric outer
536 core are cooler in the downshear-left quadrant than in other quadrants in SH15 (Fig.
537 5c). Although latent heating related to the downshear-left updrafts is visible downshear
538 left above the boundary layer (Fig. 6e) and positive horizontal advection exhibits in the
539 downshear-left midtroposphere (Fig. 9e), the vertical advection simultaneously brings
540 about enhanced adiabatic cooling above $z = 3$ km (Fig. 9g). This negative vertical
541 advection of θ surpasses the latent heating and positive horizontal advection, resulting
542 in lower θ in the downshear-left midtroposphere. Similar findings were also described
543 in Zhang et al. (2002). Nevertheless, the presence of higher mid-tropospheric θ_e
544 values in the downshear-left outer core (Fig. 7e) demonstrates that the positive
545 horizontal advection of both θ and q_v (Figs. 9e and 9f), along with the positive
546 vertical advection of q_v (Fig. 9h) near the midlevels, surpasses the influence of
547 midlevel cold air in that quadrant.

548 Although positive horizontal advection contributing to the θ_e tendency in the
549 downshear-left quadrant also exists above $z = 9$ km in SH15 (Fig. 8e), such an advective
550 contribution is weaker than that in the midtroposphere. At upper levels, the warm and
551 moist core of the TC in SH15 is advected more downshear left (Figs. 10c and 10d) than
552 in SH05 (Figs. 10a and 10b), illustrating the strong upper-level advective ventilation
553 effect by the strong asymmetric flow in SH15, as also pointed out in Fu et al. (2019).
554 Although the asymmetric outflow in the downshear-left upper layers (Figs. 10c and 10d)

555 is stronger than that near the midtroposphere (Figs. 5c and 5d), positive horizontal
556 advection of θ and q_v in the downshear-left quadrant above $z = 9$ km is smaller than
557 that around the midlevels (Figs. 9e and 9f) due to the relatively smaller horizontal
558 gradients of θ and q_v in the downshear-left upper layers (Figs. 10c and 10d). The
559 positive horizontal advective contribution to the θ_e tendency in the downshear-left
560 quadrant above $z = 9$ km is thereby less than that near the midtroposphere in SH15 (Fig.
561 8e).

562 The positive contribution by surface fluxes (Fig. 8g) offsets part of the negative
563 effects of advection in the boundary layer (Figs. 8e and 8f), but negative θ_e tendencies
564 sometimes remain evident in the downshear-left boundary layer (Fig. 8h). Downshear-
565 left θ_e is thus reduced and becomes lower in the outer-core boundary layer (Fig. 7b)
566 after the shear is introduced in SH15, compared to the θ_e values in other quadrants.
567 The negative downshear-left horizontal advective contribution to θ_e below $z = 3$ km
568 (Fig. 8e) results primarily from negative horizontal advection related to q_v (Fig. 9f).
569 In the boundary layer, lower q_v exists left-of-shear because of precipitation-forced
570 downdrafts, and the q_v value decreases radially outward (Fig. 4d). The shear triggers
571 asymmetric inflow in the downshear-left boundary layer (Fig. 4d) and is responsible
572 for negative horizontal advection related to q_v particularly between 100 and 200 km
573 from the TC center in SH15. In contrast, although minimum θ due to evaporative
574 cooling occurs in the downshear-left boundary layer between 100 and 200 km (Fig. 4c),
575 the horizontal advective contribution of θ radially averaged between 100 and 300 km

576 remains positive in that quadrant (Fig. 9e) because of larger positive θ advection
577 between 200 and 300 km. Additionally, positive vertical advection of θ (Fig. 9g) and
578 more negative vertical advection of q_v confined within the downshear-left boundary
579 layer (Fig. 9h) reflect the presence of low-level downdrafts adjacent to the convective-
580 scale updraft cores in that quadrant (Didlake and Houze 2009; Li and Wang 2012b) and
581 the downward transport of low-entropy air. Consequently, negative net vertical and
582 horizontal advective contributions (Figs. 8e and 8f) partly balance the surface fluxes,
583 resulting jointly in negative θ_e tendencies within the downshear-left boundary layer
584 in SH15 (Fig. 8h).

585 In the experiment SH25 in which a shear of 25 m s^{-1} is introduced, the θ_e budget
586 results in the downshear-left quadrant mirror those in SH15. The midlevel maximum
587 θ_e tendency is also evident in the downshear-left quadrant in SH25, and it becomes
588 much larger and deeper (Fig. 11h), compared to SH15. The enhancement of the
589 tendency at those levels is because of increases in both horizontal and vertical advection
590 (Figs. 11e and 11f), conducive to the increasing θ_e downshear left (Fig. 7f). Compared
591 to SH15, the asymmetric midlevel outflow of the TC simulated in SH25 is stronger and
592 deeper in the downshear-left quadrant (Figs. 5c, 5f, 7e, and 7f), because of much higher
593 environmental winds from the middle to upper layers in SH25. Large and deep positive
594 horizontal advection of q_v (Fig. 12f) due to the strong asymmetric outflow (Figs. 5f
595 and 7f) in the midtroposphere results in a significantly positive horizontal advective
596 contribution to the θ_e tendency there (Fig. 11e). Although the vertical advection of q_v

597 is partly counteracted by vertical advection of θ (Figs. 12g and 12h), the vertical
598 advective contribution to the θ_e tendency in the downshear-left quadrant is still
599 positive between $z = 1$ km and $z = 5.5$ km after 6 h (Fig. 11f). As a consequence, positive
600 contributions by both horizontal and vertical advection produce an enhanced positive
601 total θ_e tendency maximum at midlevels in SH25 (Fig. 11h), resulting in the
602 increasing θ_e in the downshear-left midtroposphere (Fig. 7f). Above the altitude of 9
603 km, significant positive horizontal advection is found as well in the downshear-left
604 quadrant (Fig. 11e), associated with positive horizontal advection of θ and q_v therein
605 (Figs. 12e and 12f) by significant asymmetric outflow (Figs. 7i, 10e, and 10f). On the
606 other hand, the negative θ_e tendencies become more significant in the boundary layer
607 (Fig. 11h) due mainly to the strengthening of negative vertical advection (Fig. 11f), and
608 the value of θ_e diminishes below $z = 1$ km (Fig. 7c), leading to a larger positive
609 vertical gradient of θ_e and thereby potential stability in the downshear-left quadrant
610 in SH25.

611 **5. Summary**

612 Observations have shown the presence of an azimuthally asymmetric distribution
613 of moist instability in the outer core of sheared TCs. The characteristics and associated
614 physical processes leading to the asymmetric instability are revisited in this study, based
615 on high-resolution idealized numerical simulations of weak (5 m s^{-1}), strong (15 m s^{-1})
616 and extreme (25 m s^{-1}) shear environments. The simulations demonstrate that a
617 downshear-upshear contrast in CAPE occurs in the outer core of the weakly sheared

618 TC, as found in Molinari et al. (2012), with larger (smaller) CAPE in the downshear
619 (upshear) quadrant. Potential instability at low to midlevels is also found in the
620 downshear outer core. A moderate shear (i.e., 10 m s^{-1}) simulation is also conducted,
621 and a similar downshear right–downshear left contrast in moist instability in the outer
622 core is found (not shown). As the shear magnitude increases, a more significant
623 downshear right–downshear left contrast in CAPE is observed, with larger (smaller)
624 values downshear right (left). In addition, potential instability (stability) is present
625 below (above) midlevels in the downshear-right outer core. In the downshear-left outer
626 core, there is a potentially stable environment throughout the troposphere.

627 As schematically summarized in Fig. 13, downward transports of evaporation-
628 induced, low-entropy air by convective downdrafts in the downshear-left quadrant and
629 by the mesoscale sinking motion underneath the stratiform clouds in the upshear-left
630 quadrant result in lower θ_e within the outer-core boundary layer on the left of the shear
631 vector, particularly in strongly sheared TCs. Because of the much stronger convective
632 downdrafts in the downshear-left quadrant, the lowest boundary-layer θ_e is found
633 there. The absence of sinking motion, along with near-surface fluxes, results in higher
634 θ_e within the downshear-right boundary layer in the outer core. As a result, larger
635 (smaller) CAPE occurs downshear right (left) in the outer core. An interesting feature
636 is the maximum of θ_e at midlevels in the downshear-left quadrant, which is due
637 mainly to the enhanced positive horizontal advection of θ and q_v by the shear-forced
638 asymmetric outflow (Fig. 13). As a result, the positive θ_e vertical gradient produces

639 potential stability in the outer core in the downshear-left quadrant. In contrast, the
640 negative vertical gradient of θ_e below the downshear-right midtroposphere in the
641 outer core indicates potential instability there, resulting from the surface fluxes within
642 the boundary layer and the effect of a dry intrusion at the middle to upper levels (Fig.
643 13).

644 The above findings indicate that the distribution of boundary-layer θ_e
645 considerably regulates the asymmetry in moist instability in the outer core. Many
646 studies have shown significant asymmetries in boundary-layer θ_e in the inner core of
647 sheared TCs (Riemer et al. 2010; Zhang et al. 2013; Tao and Zhang 2014, 2019). This
648 boundary-layer θ_e asymmetry in the inner core is regularly time-evolving (Riemer et
649 al. 2010; Tao and Zhang 2014, 2019). Low θ_e results from the evaporative cooling of
650 convection associated with the vortex tilt (Riemer et al. 2010; Tao and Zhang 2019),
651 and generally moves downwind due to the advection of the tangential wind (Tao and
652 Zhang 2019). The degree of the boundary-layer θ_e asymmetry in the inner core tends
653 to fade as a result of the vortex alignment due to precession and near-surface entropy
654 recovery due to surface fluxes (Tao and Zhang 2019). In contrast, low θ_e values in the
655 boundary layer in the outer core of strongly sheared TCs persist in the downshear-left
656 quadrant and the θ_e in the outer core is azimuthal-asymmetrically distributed
657 significantly throughout the simulations (Figs. 7b and 7c). Therefore, the azimuthally
658 asymmetric distribution of boundary-layer θ_e in the outer core is relevant to the quasi-
659 stationary principal rainbands in shear (Li et al. 2017), as discussed in the previous

660 sections.

661 The features of azimuthally asymmetric moist instability in the outer core of TCs
662 under environmental vertical shears with different magnitudes are underpinned by
663 numerical simulations in this study, but further observational evidence of the
664 asymmetry sensitive to shear magnitude is needed. It has been uncovered that relatively
665 azimuthally symmetric conditional instability in the outer core can be triggered by outer
666 rainbands of the TC in a stationary environment and evolves with the outer rainband
667 activity (Li and Wang 2012a). The change of such azimuthally symmetric conditional
668 instability, in turn, leads to quasi-periodic behavior of the outer rainbands, which further
669 result in quasi-periodic TC intensity change (Li and Wang 2012a). The relationship
670 between the azimuthally asymmetric moist instability in the outer core of sheared TCs
671 and the initiation and development of convection (e.g., convection in outer rainbands),
672 as well as corresponding TC intensity change, has not been well studied yet, which is
673 thus worth further elaboration. More recently, TC structure and intensity change
674 pertaining to directional environmental shear flows were examined (Nolan 2011;
675 Onderlinde and Nolan 2014, 2016; Gu et al. 2018). How the asymmetric outer-core
676 moist instability behaves in such directional shear flows deserves further investigation
677 as well.

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876 **Table caption:**

877 Table 1. Brief description of physical parameterizations in TCM4.

878 **Figure captions:**

879 Figure 1. Time series of the simulated minimum surface pressure (hPa) of TCs in SH05
880 (black line), SH15 (blue line), and SH25 (red line) after vertical shears are imposed.
881 The inset displays VWS profiles corresponding to shear magnitudes of 5, 15, and
882 25 m s⁻¹. Note that the minimum surface pressure in SH25 is only shown for the
883 first 12 hours, after which the storm in that experiment decays.

884 Figure 2. CAPE (shading; J kg⁻¹) and 3-km-height reflectivity (contours; dBZ) of the
885 TCs simulated in (a) SH05 at 24 h, (b) SH15 at 24 h, and (d) SH25 at 9 h.
886 Reflectivity is contoured at 10, 20, 30, and 45 dBZ with lighter colors indicating
887 larger values. Black dashed concentric circles are every 100 km from the TC center,
888 and shear direction is indicated by the black arrow.

889 Figure 3. Time-azimuth distributions of CAPE (top panels; J kg⁻¹) and CIN (bottom
890 panels; J kg⁻¹) radially averaged between 100 and 300 km for (a), (d) SH05, (b), (e)
891 SH15, and (c), (f) SH25. “UL”, “UR”, “DR”, and “DL” denote shear-relative
892 quadrants of upshear left, upshear right, downshear right, and downshear left,
893 respectively. Note that the results after 12 h in SH25 are excluded in (c), and (f)
894 because the modeled TC decays after that time.

895 Figure 4. θ (left column; shading; unit: K) and q_v (right column; shading; unit: g kg⁻¹)
896 vertically averaged between $z = 0.1$ and 0.96 km and temporally averaged
897 between 0 and 48 h in (a–b) SH05, (c–d) SH15, and averaged between 0 and 12 h

898 in (e–f) SH25, superposed by asymmetric winds (black vectors). Black dashed
899 concentric circles are every 100 km from the TC center, and shear direction is
900 indicated by the black arrow.

901 Figure 5. As in Fig. 4, but for quantities vertically averaged between $z = 4.3$ and 5.0
902 km. Note that scales of the color bars are different from those in Fig. 4.

903 Figure 6. Time-height cross sections of mean (radially averaged between 100 and 300
904 km) latent heating rate (K h^{-1}) in SH05 (first column), SH15 (second column), and
905 SH25 (third column). Condensational heating for the downshear-right, downshear-
906 left, upshear-left, and upshear-right quadrants are depicted in the (a–c) first, (d–f)
907 second, (g–i) third, and (j–l) fourth rows, respectively. Note that the stippling
908 indicates sinking regions.

909 Figure 7. Time-azimuth distributions of θ_e (shading; unit: K) radially averaged
910 between 100 and 300 km, superposed by asymmetric radial flows (arrows). Top
911 panels show values vertically averaged between $z = 0.1$ and 0.96 km along with
912 asymmetric inflows, middle panels show values vertically averaged between $z =$
913 4.3 and 5.0 km along with asymmetric outflows, and bottom panels display values
914 vertically averaged between $z = 9.6$ and 10.6 km along with asymmetric outflows
915 for (a), (d), (g) SH05, (b), (e), (h) SH15, and (c), (f), (i) SH25. Note that the results
916 after 12 h in SH25 are excluded in (c), (f), and (i), because the modeled TC decays
917 after that time.

918 Figure 8. Time-height cross sections of mean (radially averaged between 100 and 300
919 km) contributions to the θ_e tendency (K h^{-1}) in SH15 by the horizontal advection
920 (first column), vertical advection (second column), and diabatic processes (third
921 column). The panels in the fourth column show total θ_e tendency. Budget results
922 for the downshear-right, downshear-left, upshear-left, and upshear-right quadrants
923 are depicted in the (a–d) first, (e–h) second, (i–l) third, and (m–p) fourth rows,
924 respectively.

925 Figure 9. Time-height cross sections of mean (radially averaged between 100 and 300
926 km) contributions to the θ_e tendency (K h^{-1}) in SH15 by the horizontal advection
927 of θ (first column), horizontal advection associated with q_v (second column),
928 vertical advection of θ (third column), and vertical advection associated with q_v
929 (fourth column). Results for the downshear-right, downshear-left, upshear-left, and
930 upshear-right quadrants are depicted in the (a–d) first, (e–h) second, (i–l) third, and
931 (m–p) fourth rows, respectively.

932 Figure 10. As in Fig. 4, but for quantities vertically averaged between $z = 9.6$ and 10.6
933 km. Note that scales of the color bars are different from those in Fig. 4.

934 Figure 11. As in Fig. 8, but for SH25.

935 Figure 12. As in Fig. 9, but for SH25.

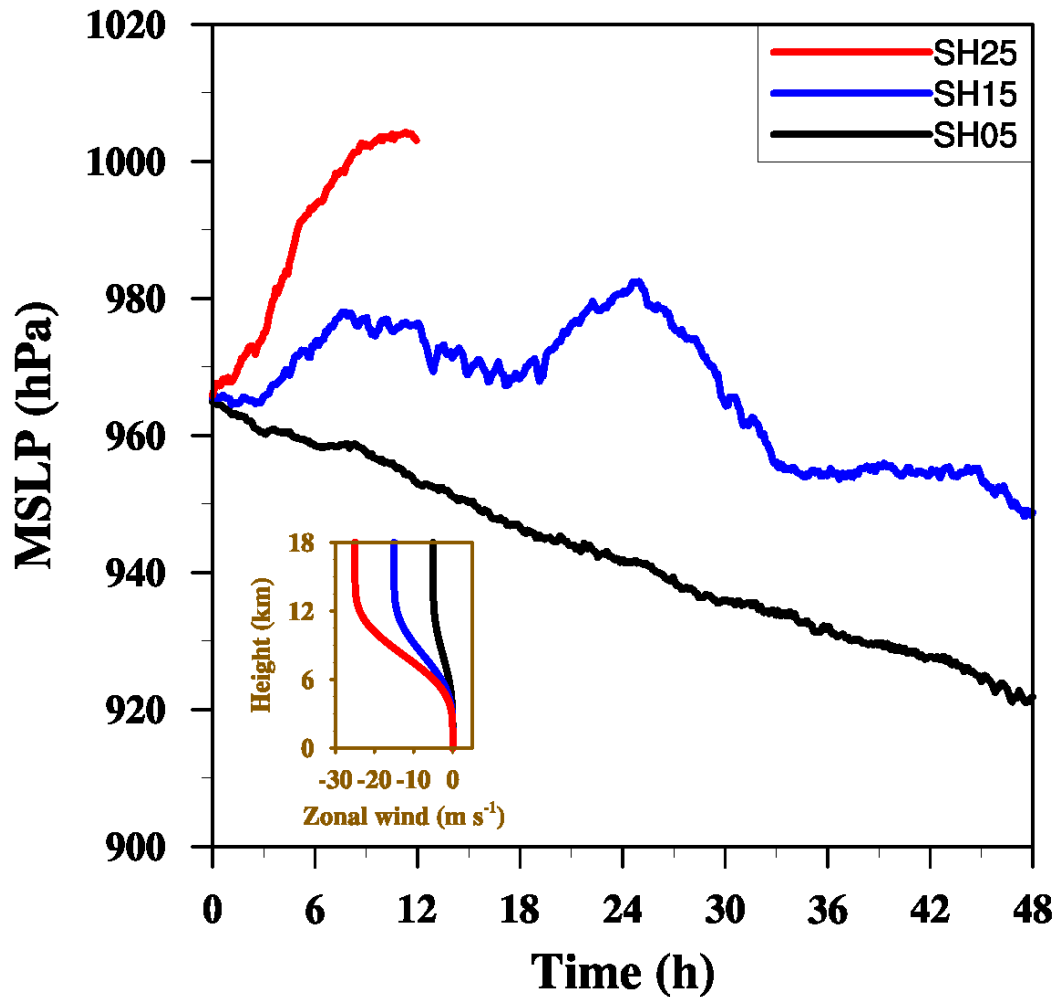
936 Figure 13. Three-dimensional schematic summarizing the processes causing

937 azimuthally asymmetric moist instability in the outer core of a sheared TC.

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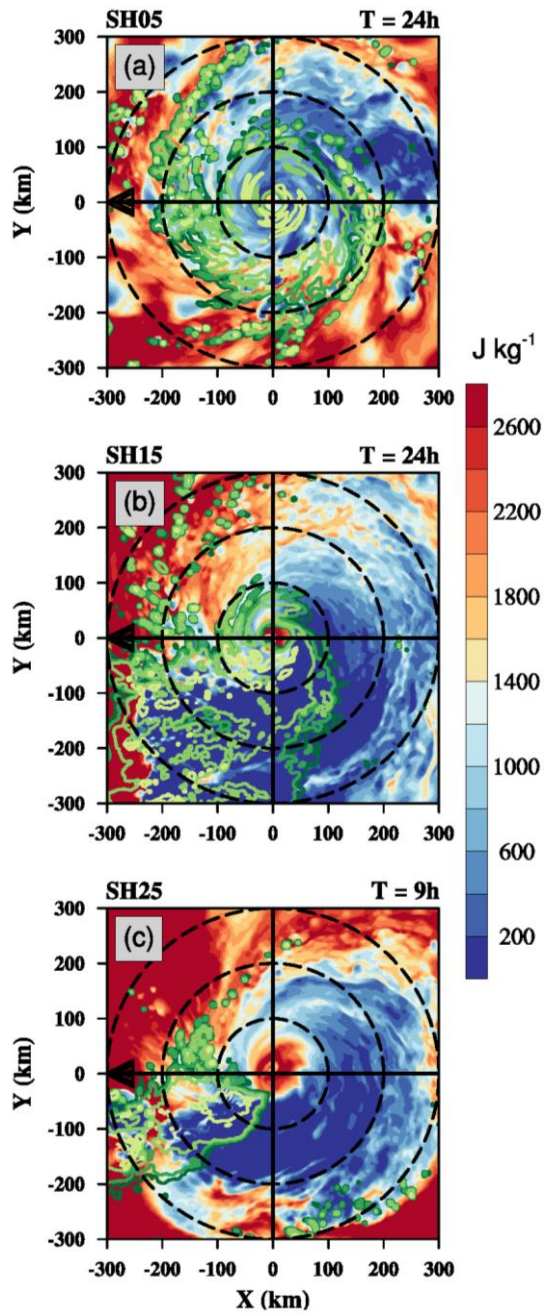
TABLE 1. Brief description of physical parameterizations in TCM4.

Basic model physics	Description
Cumulus parameterization	No
Cloud microphysics	Explicit mixed-phase cloud microphysics (Wang 2001)
Surface layer scheme	Modified Monin–Obukhov scheme (Fairall et al. 2003; Wang 2002)
Subgrid-scale vertical turbulent mixing	E – ε turbulence closure scheme (Langland and Liou 1996)
Longwave radiative cooling	A Newtonian cooling term added to the perturbation potential temperature equation (Rotunno and Emanuel 1987)



940

941 FIG. 1. Time series of the simulated minimum surface pressure (hPa) of TCs in SH05
 942 (black line), SH15 (blue line), and SH25 (red line) after vertical shears are imposed.
 943 The inset displays VWS profiles corresponding to shear magnitudes of 5, 15, and 25
 944 m s^{-1} . Note that the minimum surface pressure in SH25 is only shown for the first 12
 945 hours after, which the storm in that experiment decays.



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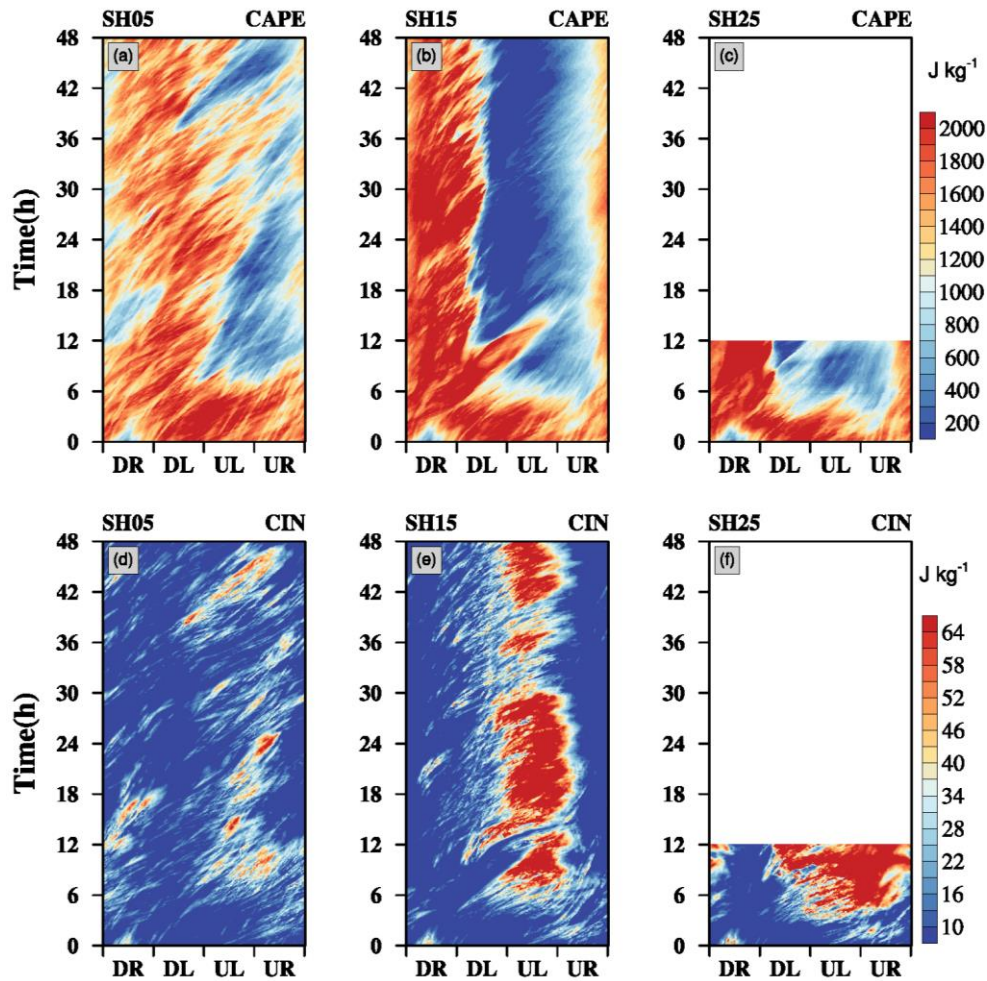
947 FIG. 2. CAPE (shading; J kg^{-1}) and 3-km-height reflectivity (contours; dBZ) of the

948 TCs simulated in (a) SH05 at 24 h, (b) SH15 at 24 h, and (d) SH25 at 9 h. Reflectivity

949 is contoured at 10, 20, 30, and 45 dBZ with lighter colors indicating larger values.

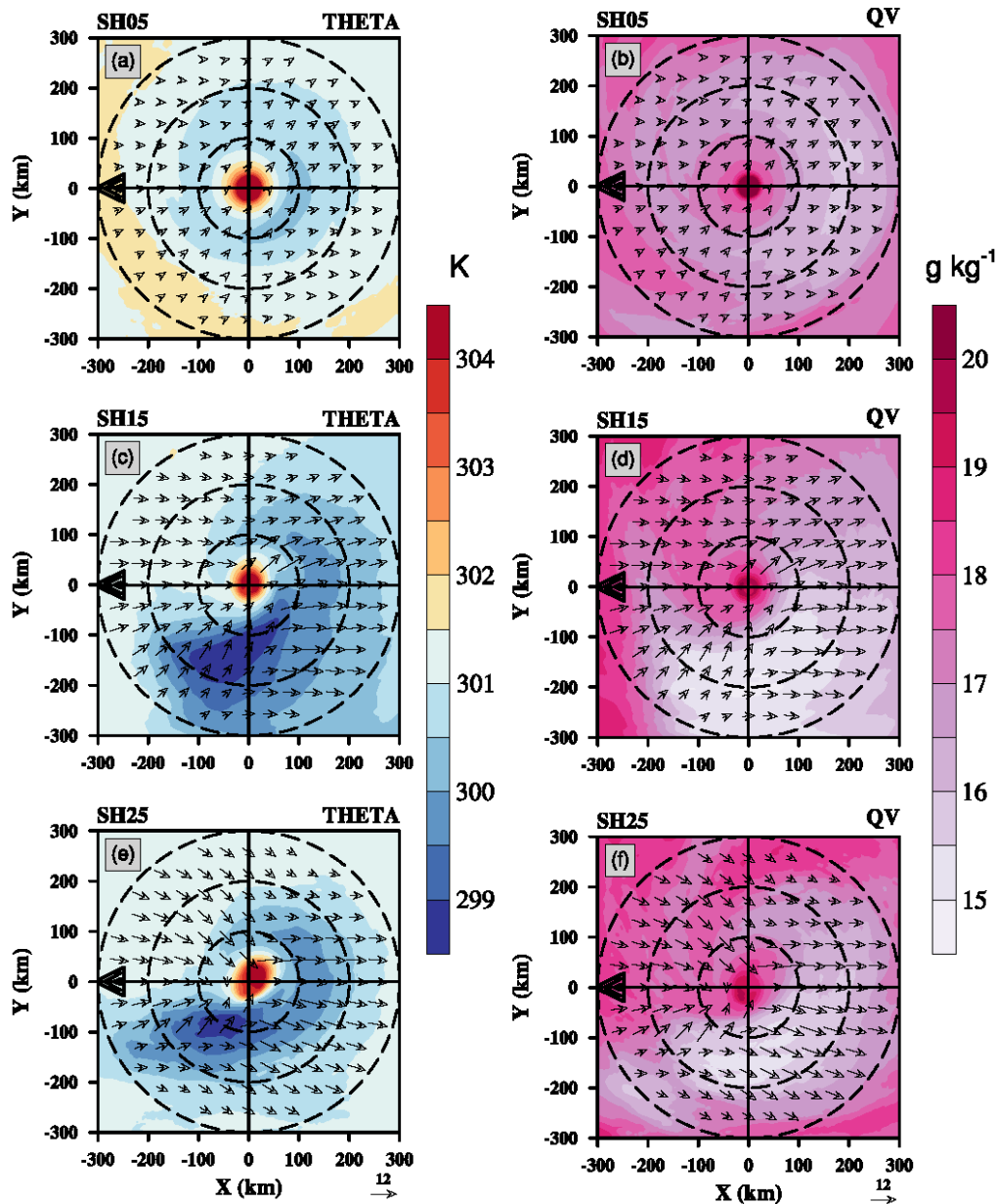
950 Black dashed concentric circles are every 100 km from the TC center, and shear

951 direction is indicated by the black arrow.



952

953 FIG. 3. Time-azimuth distributions of CAPE (top panels; J kg^{-1}) and CIN (bottom
 954 panels; J kg^{-1}) radially averaged between 100 and 300 km for (a), (d) SH05, (b), (e)
 955 SH15, and (c), (f) SH25. “UL”, “UR”, “DR”, and “DL” denote shear-relative
 956 quadrants of upshear left, upshear right, downshear right, and downshear left,
 957 respectively. Note that the results after 12 h in SH25 are excluded in (c), and (f)
 958 because the modeled TC decays after that time.



959

960 FIG. 4. θ (left column; shading; unit: K) and q_v (right column; shading; unit: g kg^{-1})

961 ¹) vertically averaged between $z = 0.1$ and 0.96 km and temporally averaged between

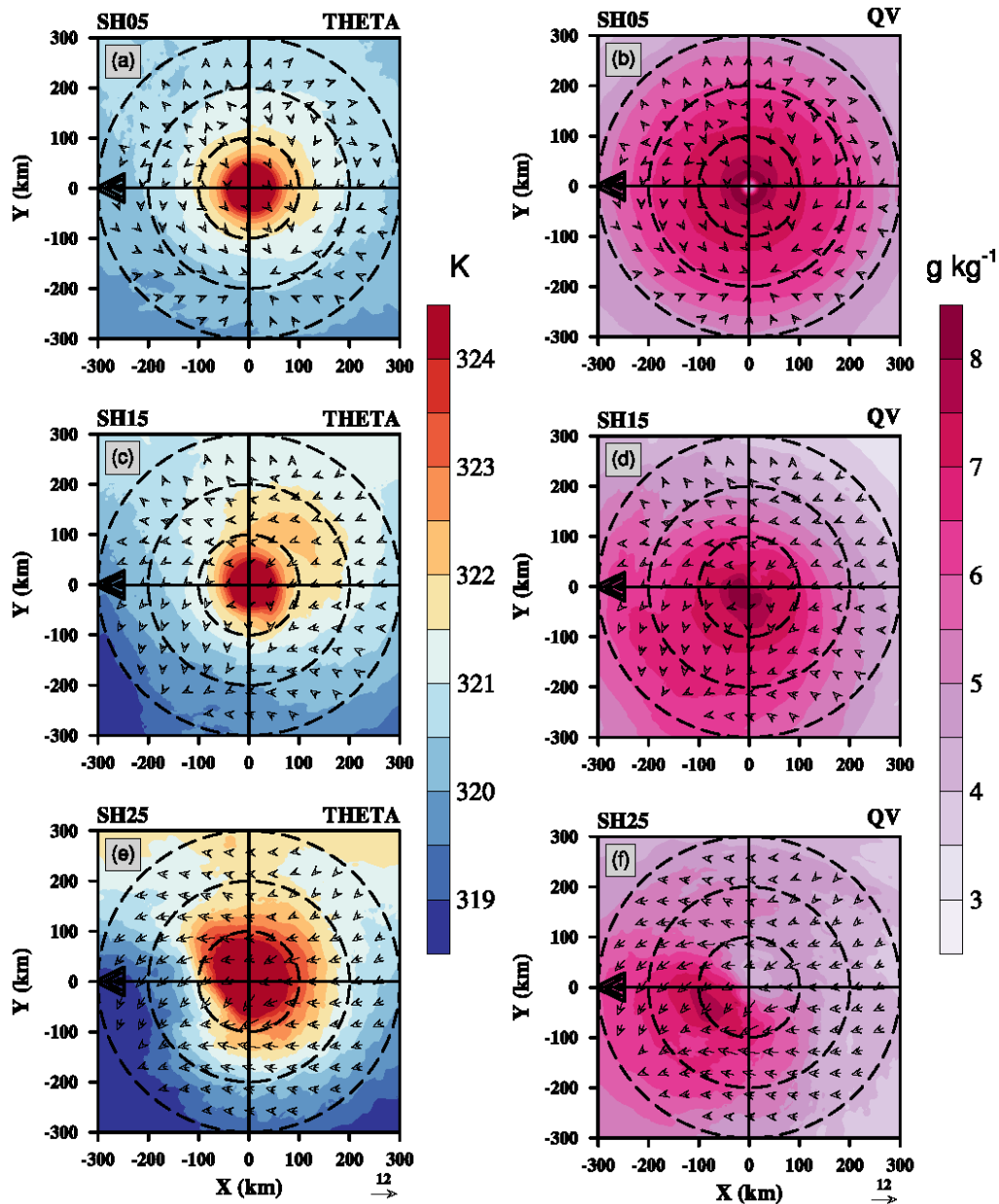
962 0 and 48 h in (a–b) SH05, (c–d) SH15, and averaged between 0 and 12 h in (e–f)

963 SH25, superposed by asymmetric winds (black vectors). Black dashed concentric

964 circles are every 100 km from the TC center, and shear direction is indicated by the

965 black arrow.

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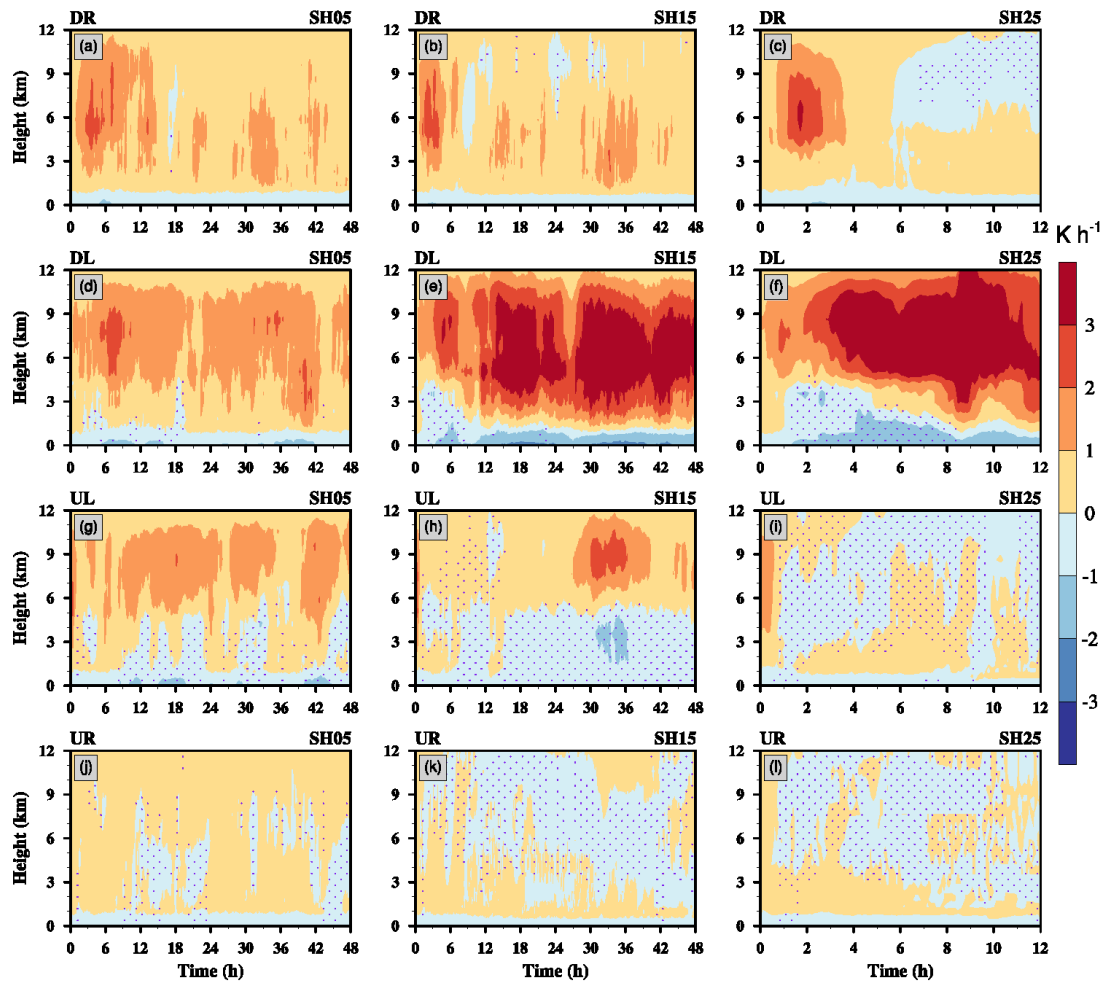
967

968 FIG. 5. As in Fig. 4, but for quantities vertically averaged between $z = 4.3$ and 5.0 km.

969

Note that scales of the color bars are different from those in Fig. 4.

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971

972 FIG. 6. Time-height cross sections of mean (radially averaged between 100 and 300

973 km) latent heating rate (K h^{-1}) in SH05 (first column), SH15 (second column), and

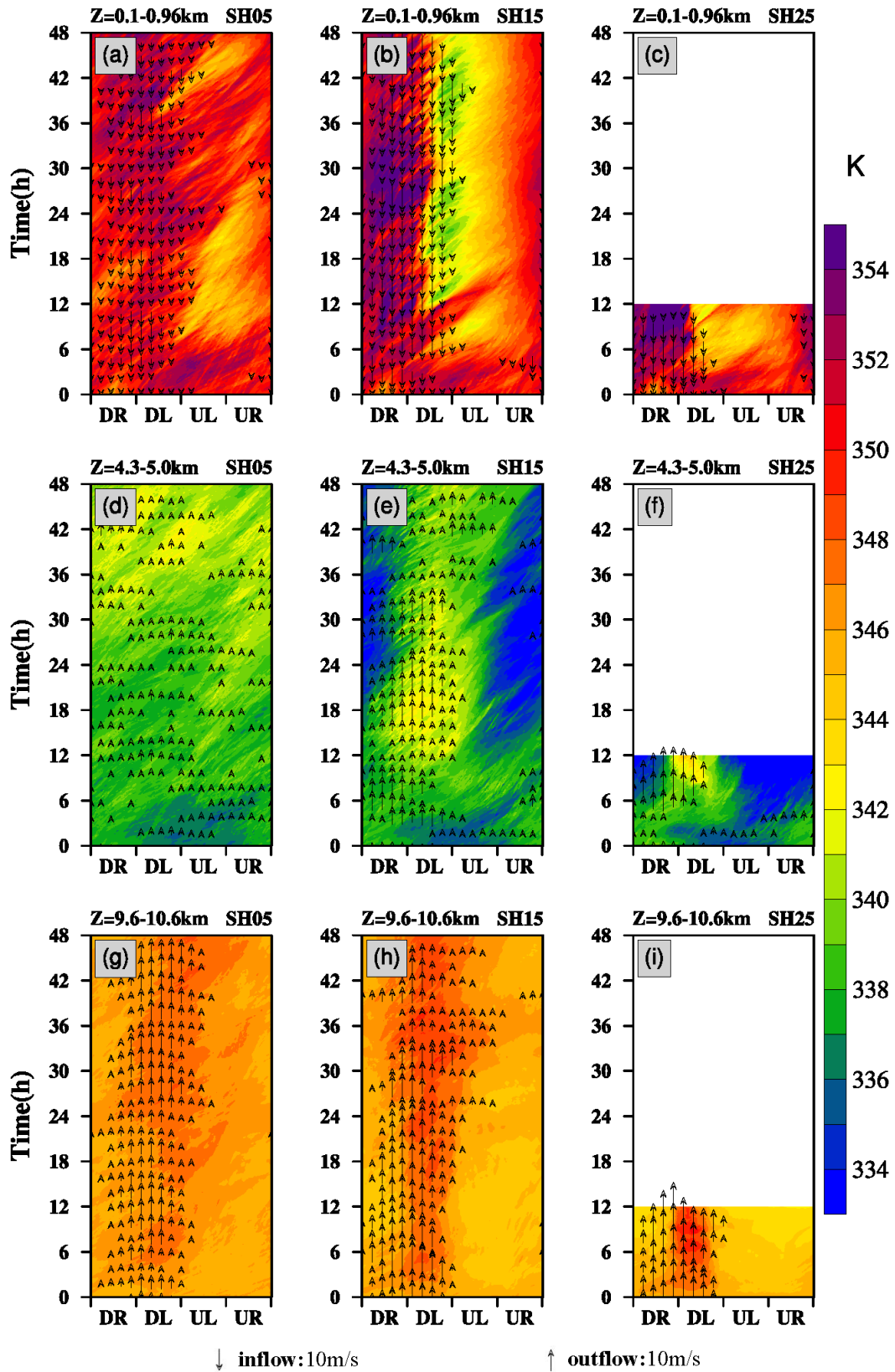
974 SH25 (third column). Condensational heating for the downshear-right, downshear-

975 left, upshear-left, and upshear-right quadrants are depicted in the (a–c) first, (d–f)

976 second, (g–i) third, and (j–l) fourth rows, respectively. Note that the stippling

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indicates sinking regions.



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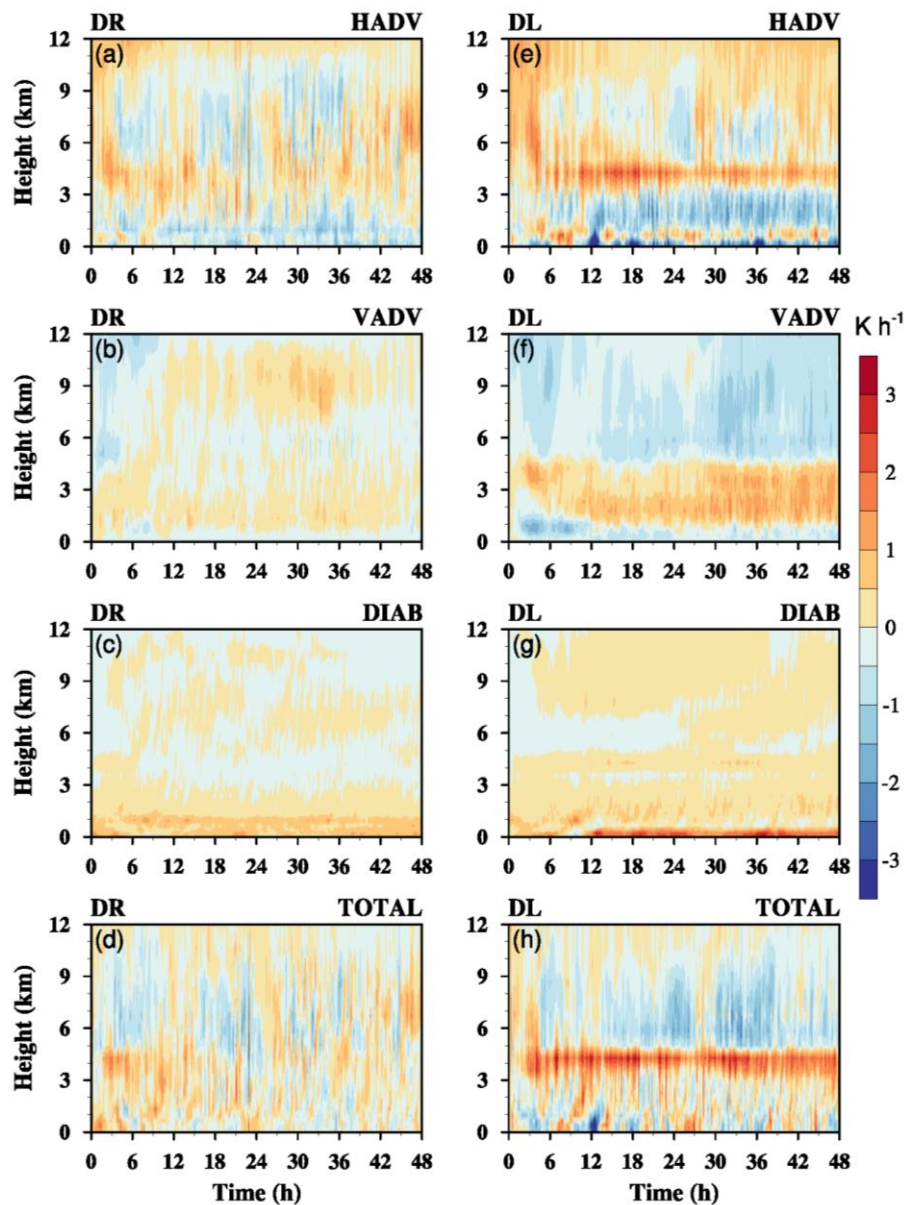
↓ inflow:10m/s

↑ outflow:10m/s

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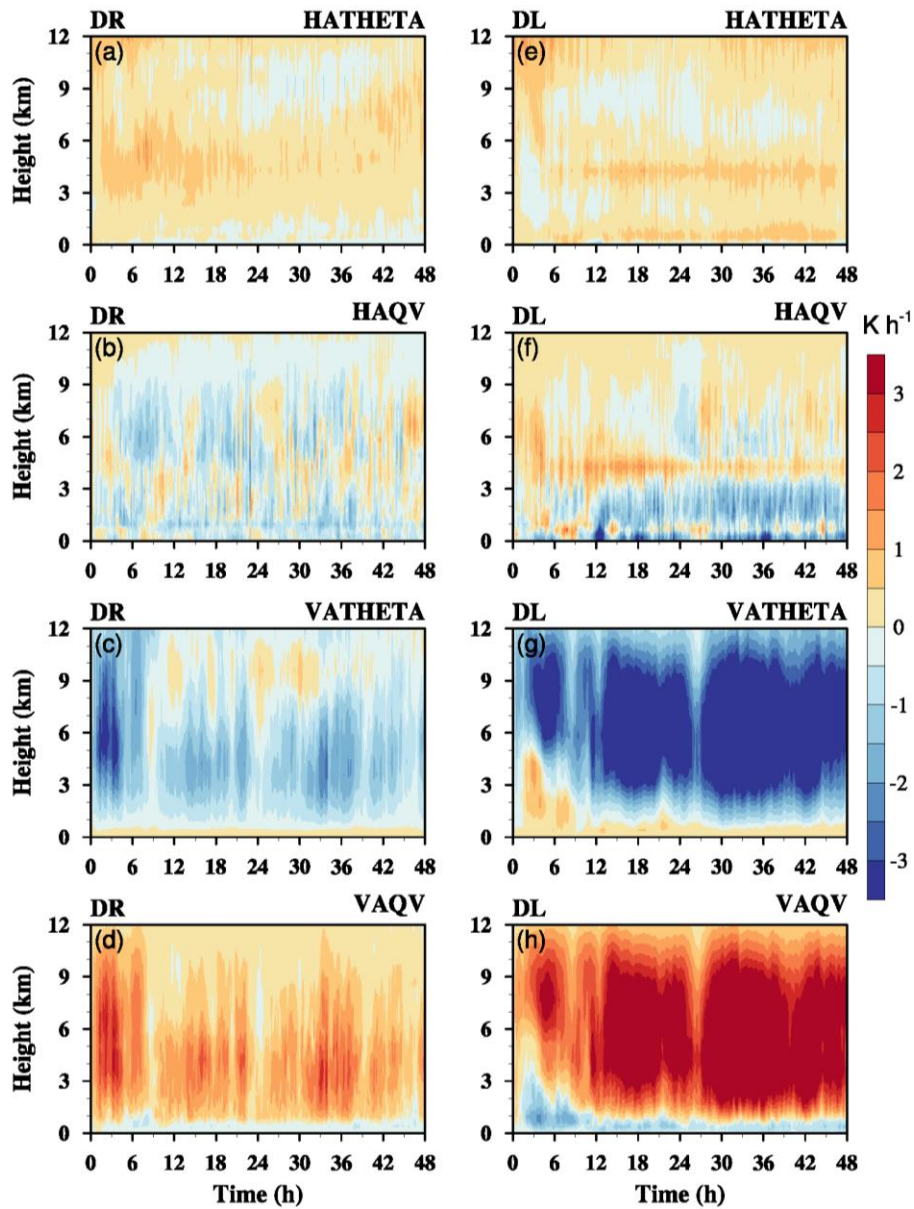
FIG. 7. Time-azimuth distributions of θ_e (shading; unite: K) radially averaged

980 between 100 and 300 km, superposed by asymmetric radial flows (arrows). Top
981 panels show values vertically averaged between $z = 0.1$ and 0.96 km along with
982 asymmetric inflows, middle panels show values vertically averaged between $z = 4.3$
983 and 5.0 km along with asymmetric outflows, and bottom panels display values
984 vertically averaged between $z = 9.6$ and 10.6 km along with asymmetric outflows for
985 (a), (d), (g) SH05, (b), (e), (h) SH15, and (c), (f), (i) SH25. Note that the results after
986 12 h in SH25 are excluded in (c), (f), and (i), because the modeled TC decays after
987 that time.



988

989 FIG. 8. Time-height cross sections of mean (radially averaged between 100 and 300
 990 km) contributions to the θ_e tendency (K h^{-1}) in SH15 by the horizontal advection
 991 (first row), vertical advection (second row), and diabatic processes (third row). The
 992 panels in the fourth row show total θ_e tendency. Budget results for the downshear-
 993 right and downshear-left quadrants are depicted in the (a–d) left and (e–h) right
 994 columns, respectively.



995

996 FIG. 9. Time-height cross sections of mean (radially averaged between 100 and 300

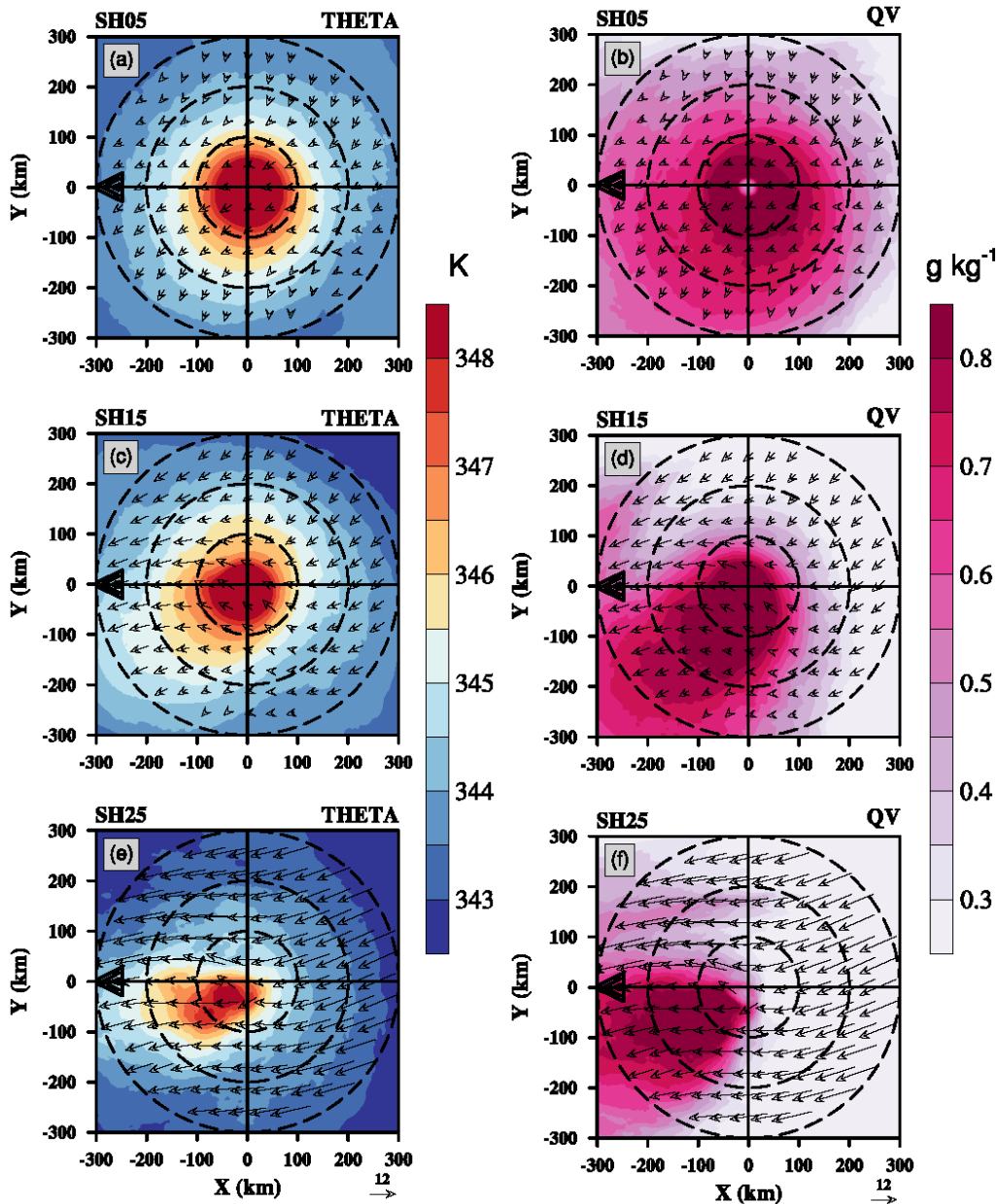
997 km) contributions to the θ_e tendency (K h^{-1}) in SH15 by the horizontal advection of

998 θ (first row), horizontal advection associated with q_v (second row), vertical

999 advection of θ (row), and vertical advection associated with q_v (fourth row).

1000 Results for the downshear-right and downshear-left quadrants are depicted in the (a–

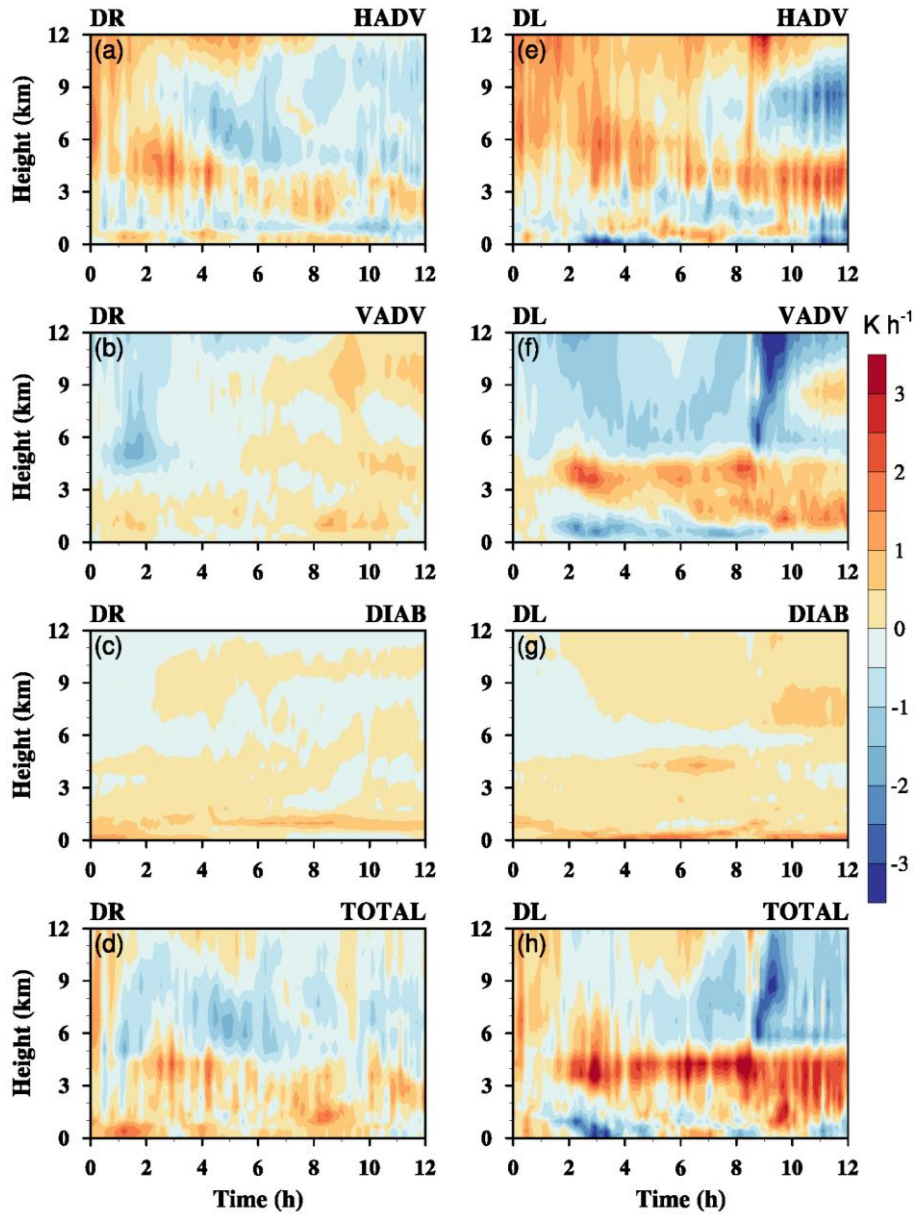
1001 d) left and (e–h) right columns, respectively.



1002

1003 FIG. 10. As in Fig. 4, but for quantities vertically averaged between $z = 9.6$ and 10.6

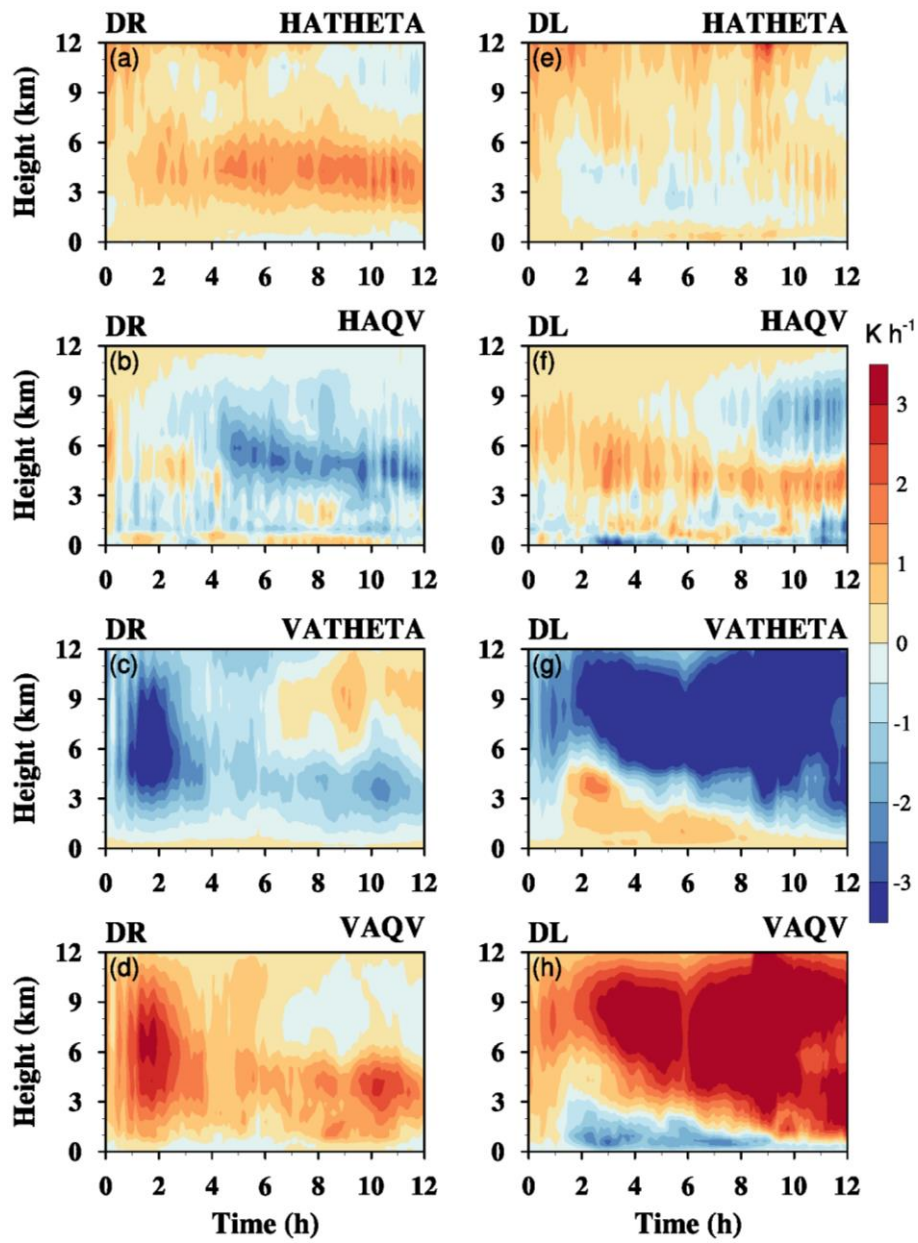
1004 km. Note that scales of the color bars are different from those in Fig. 4.



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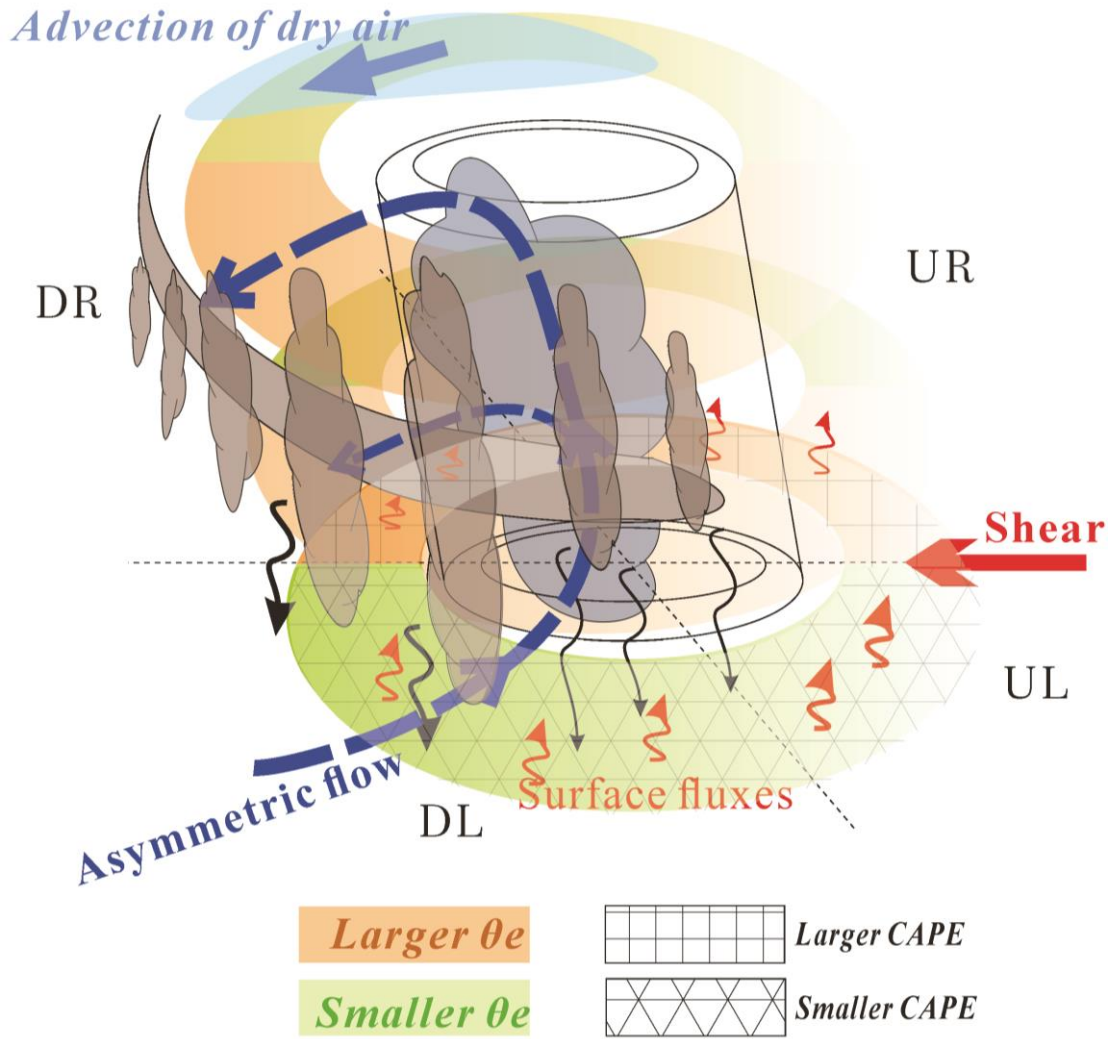
FIG. 11. As in Fig. 8, but for SH25.



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FIG. 12. As in Fig. 9, but for SH25.



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FIG. 13. Three-dimensional schematic summarizing the processes causing

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azimuthally asymmetric moist instability in the outer core of a sheared TC.