Efficient calculation of interior scattering from cavities with small modifications

Xinlei Chen✉, Xiuqiang Liu and Changqing Gu

The analysis of the interior scattering from open cavities with small modifications is an important task in designing a stealthy jet engine. Previous research has shown the magnetic field integral equation with the Kirchhoff approximation can be used to calculate the cavity interior scattering. However, it must repeat the expensive method of moments (MoM) solution even when the cavity is modified only slightly. In this Letter, the efficient method based on the partitioned-inverse formula and the Sherman–Morrison–Woodbury formula is employed to address this problem. It can avoid the repeated MoM direct solution. We only need to solve the lower-upper (LU) decomposition of the impedance matrix of the original cavity, and can efficiently derive the solution of the modified cavities via matrix identities without loss of accuracy. Numerical results are given to demonstrate the performance of the proposed approach.

Introduction: The analysis of the interior scattering from open-ended cavities is an important task in designing a stealthy jet aircraft, because the engine inlets and nozzles may be the major contributors to the overall radar cross section (RCS). The high-frequency asymptotic approaches, such as the shooting and bouncing ray [1] and the iterative physical optics [2], have been successfully applied to analyse electrically large cavities with simple terminations. However, they are not expected to provide accurate results for cavities with complex terminations [2]. The method of moments (MoM) [3] with Kirchhoff approximation (KA) [4] can be more accurate than the high-asymptotic approaches for complex cavities. In the practical application, the cavity usually is repeatedly modified and analysed to find the geometry with a better stealth effect. The conventional MoM [3] may spend a lot of CPU time for this type of problem because the impedance matrix equation must be repeatedly solved each time the cavity's geometry is modified. The objective of this Letter is to develop an efficient technique for calculating the interior scattering from a cavity with small modifications.

Efficient method for cavity with modifications: An open-ended perfect electric conducting (PEC) cavity is illuminated by an incident plane wave ($\boldsymbol{E}^i$, $\boldsymbol{H}^i$), as shown in Fig. 1. According to the equivalence principle [5], the induced electric current $\boldsymbol{J}(\boldsymbol{r})$ on the internal surfaces S_c of the cavity satisfies the magnetic field integral equation as follows [2, 3]:

$$\frac{\boldsymbol{J}(\boldsymbol{r})}{2} - \hat{\boldsymbol{n}}(\boldsymbol{r}) \times \nabla \times \oint_{S_c} \boldsymbol{J}(\boldsymbol{r}')\nabla G(\boldsymbol{r}, \boldsymbol{r}')\,\mathrm{d}s' = \hat{\boldsymbol{n}}(\boldsymbol{r}) \times \boldsymbol{H}_a^i(\boldsymbol{r}) \tag{1}$$

where $\hat{\boldsymbol{n}}(\boldsymbol{r})$ is the unit normal vector pointing into the cavity, $G(\boldsymbol{r}, \boldsymbol{r}')$ is the Green's function in free space, and $\oint$ denotes the principal value of the integral. $\boldsymbol{H}_a^i(\boldsymbol{r})$ denotes the incident magnetic field on S_c. Using the KA [2, 4], it is approximately equal to

$$\boldsymbol{H}_a^i(\boldsymbol{r}) \approx -\int_{S_a} \boldsymbol{J}^i(\boldsymbol{r}') \times \nabla G(\boldsymbol{r}, \boldsymbol{r}')\,\mathrm{d}s' - \frac{1}{\mathrm{j}k\eta}\nabla \times \int_{S_a} \boldsymbol{M}^i(\boldsymbol{r}') \times \nabla G(\boldsymbol{r}, \boldsymbol{r}')\,\mathrm{d}s' \tag{2}$$

where k and η are the wavenumber and impedance in free space. $\boldsymbol{J}^i(\boldsymbol{r})$ and $\boldsymbol{M}^i(\boldsymbol{r})$ are the equivalent electric and magnetic current on the aperture S_a of the cavity. They are induced by the incident plane wave

$$\boldsymbol{J}^i(\boldsymbol{r}) = \hat{\boldsymbol{n}}(\boldsymbol{r}) \times \boldsymbol{H}^i(\boldsymbol{r}), \quad \boldsymbol{M}^i(\boldsymbol{r}) = \boldsymbol{E}^i(\boldsymbol{r}) \times \hat{\boldsymbol{n}}(\boldsymbol{r}). \tag{3}$$

Using the method of moments (MoM), the integral equation (1) can be discretised into the impedance matrix equation as follows:

$$\boldsymbol{Z}_{cc}\boldsymbol{I}_c = \boldsymbol{V}_c. \tag{4}$$

The details of the elements of $\boldsymbol{Z}_{cc}$ and $\boldsymbol{V}_c$ can be found in [6].

When designing a stealthy cavity, we usually analyse the scattering from the cavity with small modifications. For example, the termination of a cavity is changed, as shown in Figs. 2*a* and *b*. The conventional MoM is very expensive for the application, because (4) has to be solved each time the cavity is modified. In this Letter, the problem is mitigated by reusing the lower-upper (LU) decomposition of the impedance matrix $\boldsymbol{Z}_{cc}$ of the original cavity.

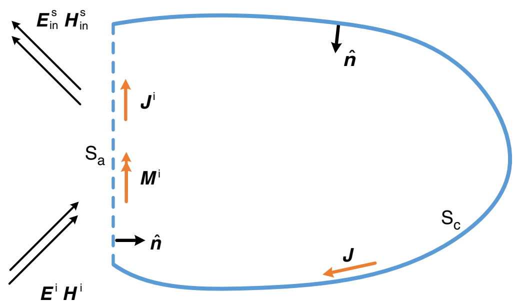

Fig. 1 *Open-ended cavity is illuminated by a plane wave*

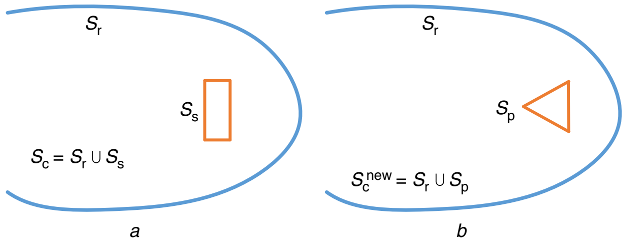

Fig. 2 *Original and modified cavities. Termination is changed from cylinder into cone, while cavity wall remains unchanged*

a Original cavity whose termination is cylinder
b Modified cavity whose termination is cone

The partial modification of the cavity in Fig. 2 can be divided into two steps: first subtracting S_s from the original cavity S_c and then adding S_p to the rest structure S_r. Correspondingly, the matrix equations for the original and modified cavities can be partitioned into [7]

$$\begin{bmatrix} \boldsymbol{Z}_{rr} & \boldsymbol{Z}_{rs} \\ \boldsymbol{Z}_{sr} & \boldsymbol{Z}_{ss} \end{bmatrix}\begin{bmatrix} \boldsymbol{I}_r \\ \boldsymbol{I}_s \end{bmatrix} = \begin{bmatrix} \boldsymbol{V}_r \\ \boldsymbol{V}_s \end{bmatrix}, \tag{5}$$

$$\begin{bmatrix} \boldsymbol{Z}_{rr} & \boldsymbol{Z}_{rp} \\ \boldsymbol{Z}_{pr} & \boldsymbol{Z}_{pp} \end{bmatrix}\begin{bmatrix} \boldsymbol{I}_r \\ \boldsymbol{I}_p \end{bmatrix} = \begin{bmatrix} \boldsymbol{V}_r \\ \boldsymbol{V}_p \end{bmatrix}. \tag{6}$$

By applying the partitioned-inverse formula [8] and the Sherman–Morrison–Woodbury formula [8] to (5) and (6), the electric current in the modified cavity can be efficiently computed by [7]

$$\boldsymbol{I}_p = \boldsymbol{Y}_{pp}(\boldsymbol{V}_p - \boldsymbol{Z}_{pr}\boldsymbol{Z}_{rr}^{-1}\boldsymbol{V}_r), \tag{7}$$

$$\boldsymbol{I}_r = \boldsymbol{Z}_{rr}^{-1}(\boldsymbol{V}_r - \boldsymbol{Z}_{rp}\boldsymbol{I}_p), \tag{8}$$

where

$$\boldsymbol{Y}_{pp} = (\boldsymbol{Z}_{pp} - \boldsymbol{Z}_{pr}\boldsymbol{Z}_{rr}^{-1}\boldsymbol{Z}_{rp})^{-1}. \tag{9}$$

To efficiently calculate $\boldsymbol{Z}_{rr}^{-1}\boldsymbol{V}_r$, $\boldsymbol{Z}_{rr}^{-1}(\boldsymbol{V}_r - \boldsymbol{Z}_{rp}\boldsymbol{I}_p)$, and $\boldsymbol{Z}_{rr}^{-1}\boldsymbol{Z}_{rp}$ in (7)–(9), we use (10) rather than directly perform the LU decomposition of $\boldsymbol{Z}_{rr}$ [7]

$$\boldsymbol{Z}_{rr}^{-1}\boldsymbol{A} = (\boldsymbol{Y}_{rr}\boldsymbol{A}) + \boldsymbol{Y}_{rs}(\boldsymbol{1} - \boldsymbol{Z}_{sr}\boldsymbol{Y}_{rs})^{-1}\boldsymbol{Z}_{sr}(\boldsymbol{Y}_{rr}\boldsymbol{A}), \tag{10}$$

because the size of $\boldsymbol{1} - \boldsymbol{Z}_{sr}\boldsymbol{Y}_{rs}$ is much smaller than the size of $\boldsymbol{Z}_{rr}$.

$\boldsymbol{Y}_{rr}\boldsymbol{A}$ and $\boldsymbol{Y}_{rs}$ in (10) can be efficiently obtained by using the LU decomposition of $\boldsymbol{Z}_{cc}$ as follows:

$$\begin{bmatrix} \boldsymbol{Y}_{rs} \\ \boldsymbol{Y}_{ss} \end{bmatrix} = \boldsymbol{Z}_{cc}^{-1}\begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{1} \end{bmatrix}, \quad \begin{bmatrix} \boldsymbol{Y}_{rr}\boldsymbol{A} \\ \boldsymbol{Y}_{sr}\boldsymbol{A} \end{bmatrix} = \boldsymbol{Z}_{cc}^{-1}\begin{bmatrix} \boldsymbol{A} \\ \boldsymbol{0} \end{bmatrix}, \tag{11}$$

because $\boldsymbol{Y}_{rr}$ and $\boldsymbol{Y}_{rs}$ are the submatrices of the inverse of $\boldsymbol{Z}_{cc}$

$$\begin{bmatrix} \boldsymbol{Y}_{rr} & \boldsymbol{Y}_{rs} \\ \boldsymbol{Y}_{sr} & \boldsymbol{Y}_{ss} \end{bmatrix} = \boldsymbol{Z}_{cc}^{-1} = \begin{bmatrix} \boldsymbol{Z}_{rr} & \boldsymbol{Z}_{rs} \\ \boldsymbol{Z}_{sr} & \boldsymbol{Z}_{ss} \end{bmatrix}^{-1}. \tag{12}$$

We should note that $\boldsymbol{1}$ and $\boldsymbol{0}$ denote the identity matrix and zeros matrix.

After getting the electric current $\boldsymbol{J}(\boldsymbol{r})$ on the internal surfaces of the cavity, the electromagnetic scattering from the cavity interior can be computed by using the KA again [2, 4]

$$\boldsymbol{E}_{\text{in}}^{\text{s}}(\boldsymbol{r}) \approx \int_{S_a} \boldsymbol{M}^{\text{s}}(\boldsymbol{r}') \times \nabla G(\boldsymbol{r}, \boldsymbol{r}')\,\text{d}s' - \frac{\eta}{\text{j}k}\nabla \times \int_{S_a} \boldsymbol{J}^{\text{s}}(\boldsymbol{r}') \times \nabla G(\boldsymbol{r}, \boldsymbol{r}')\,\text{d}s' \tag{13}$$

where $\boldsymbol{J}^{\text{s}}(\boldsymbol{r})$ and $\boldsymbol{M}^{\text{s}}(\boldsymbol{r})$ are the equivalent electric and magnetic current on the aperture induced by the scattering field $\boldsymbol{E}^{\text{s}}(\boldsymbol{r})$ and $\boldsymbol{H}^{\text{s}}(\boldsymbol{r})$ from electric current on the internal surfaces of the cavity

$$\boldsymbol{J}^{\text{s}}(\boldsymbol{r}) = -\hat{\boldsymbol{n}}(\boldsymbol{r}) \times \boldsymbol{H}^{\text{s}}(\boldsymbol{r}) = \hat{\boldsymbol{n}}(\boldsymbol{r}) \times \int_{S_c} \boldsymbol{J}(\boldsymbol{r}') \times \nabla G(\boldsymbol{r}, \boldsymbol{r}')\,\text{d}s', \tag{14}$$

$$\begin{aligned}\boldsymbol{M}^{\text{s}}(\boldsymbol{r}) &= -\boldsymbol{E}^{\text{s}}(\boldsymbol{r}) \times \hat{\boldsymbol{n}}(\boldsymbol{r}) \\ &= \frac{\eta}{\text{j}k}\left(\nabla \times \int_{S_c} \boldsymbol{J}(\boldsymbol{r}') \times \nabla G(\boldsymbol{r}, \boldsymbol{r}')\,\text{d}s'\right) \times \hat{\boldsymbol{n}}(\boldsymbol{r}).\end{aligned} \tag{15}$$

Numerical results: All computations are carried out on the personal computer with Intel Core i9-9900k CPU. All cavity openings are facing $\hat{z}$-axis, and the $\boldsymbol{\theta}$-polarised incident plane wave is at 10 GHz. First, we calculate a cylindrical cavity to validate our code by comparing the result with the modal solution [2]. The diameter and height of the cavity are 12 cm, as shown in the inset of Fig. 3. The cavity with the aperture is discretised into 4450 triangles. We can see from Fig. 3 that the RCS computed by our code agrees well with the modal solution [2].

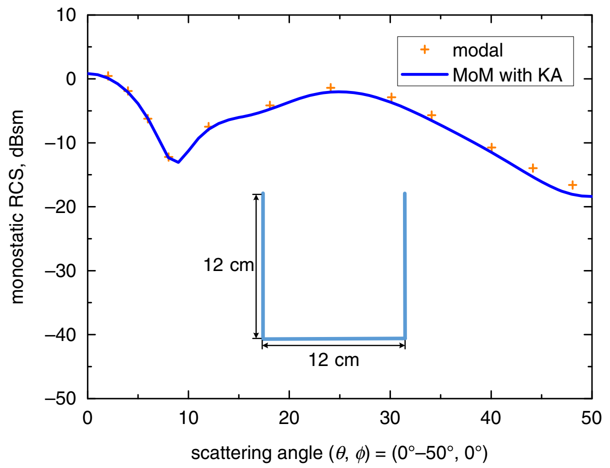

Fig. 3 *Monostatic RCS of* 4λ × 4λ *PEC cylindrical cavity compared with the modal solution for validating the code*

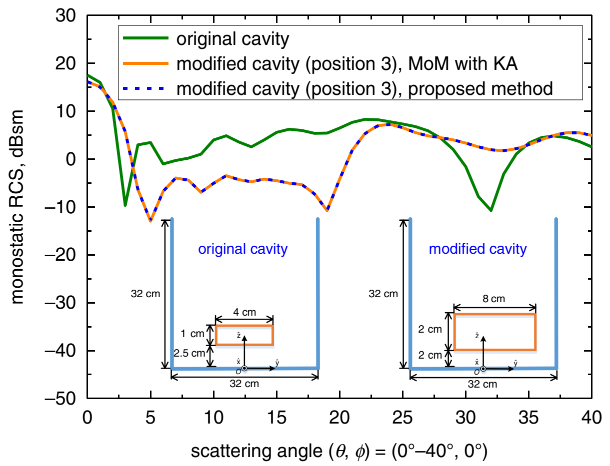

Fig. 4 *Monostatic RCS of* 10.7λ × 10.7λ *PEC cylindrical cavity with different terminations*

Next, the monostatic RCS of a PEC cylindrical cavity with different cylindrical terminations is analysed. The size of the cavity wall, original termination, and the modified termination can be found in Fig. 4. The modified terminations have the same shape but different positions. The positions of the bottom circle centres of the modified terminations are (unit: cm): (0, 0, 2), (10, 0, 2), (−10, 0, 2), (0, 10, 2), and (0, −10, 2). The aperture has 5429 triangles; the cavity wall and the original termination are modelled by 39485 and 420 RWG, respectively; the new termination has about 1542 RWG. Fig. 4 compares RCS results computed by the proposed method and the conventional MoM, and good agreement can be found. The required CPU time is summarised in Table 1. The proposed method requires the same time for generating the voltage and impedance matrix, and solving the original cavity, compared with the conventional MoM. However, the solve-time for the modified cavities is reduced by a factor of 6.

Table 1: CPU time for cavity with different modifications

	MoM with KA	This work
voltage and impedance matrix generation	21 min	
original cavity calculation	23 min	
modified cavity calculation	126 min	21 min

Conclusion: An efficient method has been presented and applied to compute the interior scattering from a cavity with modifications in this Letter. The proposed method only needs to perform the LU decomposition of the original cavity and can reuse it to efficiently obtain the solution of the modified cavities. We have shown that the proposed method has a significant improvement in the computational time for analysing a cavity with small modifications in comparison with the conventional MoM.

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One or more of the Figures in this Letter are available in colour online.

Xinlei Chen, Xiuqiang Liu and Changqing Gu (*College of Electronic and Information Engineering, Nanjing University of Aeronautics and Astronautics, People's Republic of China*)

✉ E-mail: chenxl@nuaa.edu.cn

Xinlei Chen: Also with KLME & CIC-FEMD, NUIST, People's Republic of China

Xinlei Chen: Also with State Key Laboratory of Millimeter Waves, Southest University, People's Republic of China

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